DIGITAL CATCHMENT MODEL

BASED ON SUBSURFACE FLOW

A Thesis Presented for the Degree of

Doctor of Philosophy

in Civil Engineering

in the University of Canterbury,

Christchurch, New Zealand

by M. J. Wood

1973

ABSTRACT

This thesis concerns the use of mathematical models of the rainfall-riverflow process to simulate peak flows on hydrologically small catchments. It seeks to determine whether a more detailed description of the subsurface zone of a catchment will improve the performance of such a model.

A numerical solution to Richards' equation for flow in an unsaturated porous medium applied to a one-dimensional, vertical column of soil was used to replace the subsurface components, including infiltration, of the Stanford Watershed Model. The performance of this Amended Model was compared with that of the Stanford Model itself, using rainfall and riverflow data from three New Zealand catchments. Published solutions to Richards' equation in two dimensions were also examined to estimate the usefulness of extending this approach.

The amendments reduced the number of fitted parameters from five to four, while the performance of the Amended Model proved to be comparable to that of the Stanford Model in spite of restrictions imposed by the one-dimensional formulation. The two-dimensional solutions to Richards' equation were found to offer more versatility of behaviour and physical relevance than the one-dimensional solution. It was therefore concluded that a model based on the two-dimensional solution would be superior to the Stanford Model, and that more attention should be paid to the subsurface processes by model-builders.

ACKNOWLEDGEMENTS

This work was carried out in the Civil Engineering Department of the University of Canterbury, under the overall guidance of Professor H. J. Hopkins, Head of Department.

The writer expresses his appreciation to Dr. A. J. Sutherland, who initiated the project and supervised its execution. His assistance and enthusiasm played no small part in the completion of the work. Professor I. R. Wood made many suggestions during the latter half of the project, and his experience was invaluable during the writing stage.

The staff of the Computer Centre of the University of Canterbury significantly aided the development and testing of the computer programs which formed a large part of the work, and also processed a large quantity of data. The principle sources of the data used were the Water and Soil Division of the Ministry of Works (Dr. R. P. Ibbitt), the Christchurch Hydrological Survey of the Ministry of Works (Mr. G. Martin) and the Nelson Catchment Board (Mr. D. McGowan). The assistance of these people is gratefully acknowledged.

Financial assistance from the University Grants Committee and the University of Canterbury supported the writer during the project.

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A	Constant in Philip's Infiltration Equation (Section 2.3)
A, B, C	Constants in Three-parameter Functions Representing the Hydraulic Conductivity or Moisture Content Variation (Section 5.4)
A, B, C, D	Coefficients in the Simultaneous Equations for the One-dimensional Solution to Richards' Equation (Section 5.4)
C	Specific Moisture Capacity (Section 4.2)
D	Diffusivity
E	Daily Evapotranspiration Volume
e	Evapotranspiration Rate
H	Average Depth of Subsurface Flow
h	Hydraulic Head
h_{COL}	Height of the Soil Column
h_{WT}	Height of the Water Table above the Bottom of the Soil Column
i	Potential Gradient in Darcy's Equation (Section 2.1)
i	Infiltration Rate in Philip's Equation (Section 2.3)
K	Hydraulic Conductivity
$K_{SAT,H}$	Saturated Hydraulic Conductivity in the Horizontal Direction
$K_{SAT,\ V}$	Saturated Hydraulic Conductivity in the Vertical Direction
L	Catchment Slice Length
P	Rainfall Volume in a given Time Interval
p	Rainfall Rate (Section 2.3)
p	Gauge Pressure (Section 4.1)
q	Flow Rate
q_H	Lateral Subsurface Flow per Unit Width
q_V	Sum of Soil Column Outputs per Unit Width

S	Constant in Philip's Infiltration Equation (Section 2.3)
S	Average Surface Slope (Section 5.4)
s	General Direction Ordinate
t	Time
U	Unitgraph Ordinate
v	Subsurface Velocity, defined as Discharge per Unit Area
v_{LB}	Subsurface Velocity at the Lower Boundary of the Soil Column
v_{UB}	Subsurface Velocity at the Upper Boundary of the Soil Column
v_S	Subsurface Velocity in the s Direction
v_Z	Subsurface Velocity in the z Direction
z	Elevation Head
γ	Specific Weight of Water
θ	Volumetric Moisture Content
ψ	Pressure Head (when Positive) or Soil Suction (when Negative)
ψ_j^t	Pressure-Suction at time step t and Distance Step j
τ	Time

CHAPTER ONE

INTRODUCTION

1.1 Context

Engineers who are concerned with natural waterways are concerned with flow rates, which by nature are extremely variable. Hence, although information gained by observation over a short period can be useful for determining average conditions, it is poor for determining the extremes of flow, both high and low. These extremes are the flows which govern the design of engineering works associated with the waterway, so it is necessary to interpret these short-term observations in the light of an understanding of the physical processes.

Since Palissey[1] correctly postulated in 1580 that riverflow was derived from rainfall, investigators have been seeking to understand and quantify the relation between them. They have defined the "zone of influence" for flow at a point on a river as the catchment area enclosed within the topographic divides, but the nature of the function which translates input to the system (precipitation on the catchment) into output (riverflow) is difficult to describe generally.

One way of describing this function is by building a model. A model is an analogue which duplicates certain characteristics of the system but is easier to operate and measure. If a model can be built which describes the rainfall-riverflow function sufficiently accurately, it can be used to forecast riverflows resulting from a recent or imminent rainfall, to fill in gaps in riverflow records, or to predict the effects of possible changes to the catchment. In addition the construction of a successful model is a test of the understanding of the modelled process.

Two major problems hinder the builder of a catchment model. Firstly, it is impossible to specify or even measure all the relevant catchment properties as they vary over the area of a catchment. Modellers are forced to use average, or effective, values of catchment properties, and these are difficult to estimate. Secondly, the physical processes which determine the progress of water from precipitation to riverflow are not perfectly understood. However, information about individual parts of the process, such as infiltration or interception, is constantly being discovered. This thesis is concerned with the incorporation of a revised description of one of these parts into an existing model of the whole system.

1.2 Scope

A. Model Type

Catchment behaviour can be modelled either by physical systems (scale or analogue models), or by equations and other mathematical functions (mathematical models). With so many differing processes involved, scale models have difficulty in satisfying similarity criteria. Although electrical analogues are good for solving differential equations, thay have seldom been used in hydrology. In contrast, since the advent of the digital computer many mathematical models of catchment behaviour have been constructed. It was decided at the outset to restrict this study to mathematical models.

B. Catchment Behaviour

The catchment properties which control high flows are not necessarily the same as the properties which control low flows. Since modelling necessarily involves simplification, a choice of which aspect is to receive the major attention must be made. The emphasis in this study is placed on the simulation of peak flows. This choice focusses attention onto modelling individual storms rather than continuous long-term simulation, and will require techniques which can follow rapid changes in the system.

High riverflows may be caused by two types of precipitation: rainfall and snow. The modelling of riverflows caused by melting snow would require knowledge of the energy available for melting as well as the rate of snowfall, which complicates an already complex task. Consideration of riverflows influenced by snowmelt is excluded from this study.

C. Catchment Size

A distinction is commonly made between catchments that are (hydrologically) "large" and "small". In a small catchment the channel system is very short, so that the rainfall-riverflow behaviour depends on the land characteristics such as vegetation, soil type and surface slope. In a large catchment on the other hand, the effects of the land characteristics are masked by the properties of the channel network, which therefore dominates the behaviour.

If the input to the channel system of a large catchment can be determined, modelling would involve solving the flow equations in a network of reasonably regular channels subject to lateral inflow. This solution, known as "flood routing", can be carried out mathematically with little difficulty[2]. Modelling a small catchment, however, involves consideration of the many possible mechanisms for the transfer of water from raindrop to river channel; this presents a much more challenging problem. Moreover the same considerations will determine the inputs to the channel system of a large catchment. <u>This study is restricted to catchments small enough that the channel system may be ignored.</u>

D. Study Area

The traditional view of the response of a catchment to heavy rainfall sees infiltration as a "loss" which occurs at a decreasing rate with time. Rainfall in excess of infiltration is assumed to provide the storm riverflow, but many of the methods used for translating the time distribution of this excess into a flow hydrograph imply that this is surface water, spread more or less uniformly over the catchment. On many catchments particularly in humid areas, catchment-wide overland flow does not occur even in the most intense rainfalls[3,4]. It would be desirable when modelling these catchments to include a more detailed description of the role the subsurface zone plays in determining riverflow.

A mathematical description of the flow of water in a porous medium whether saturated or unsaturated is available in Richards' equation[5]. This study seeks to determine whether the description of the subsurface zone by Richards' equation would improve the ability of a mathematical model to simulate peak flows on hydrologically small catchments.

1.3 Thesis Structure

The remainder of this thesis is divided into two parts:

(a) Chapters 2-4: Current Knowledge

Chapter Two presents a discussion of catchment modelling including methods, input and output requirements, classification and some representative examples. Chapter Three examines the ways in which catchment models may be used, leading to the choice of the Stanford Watershed Model (SWM) as the most appropriate to accept an improved description of the subsurface zone. The SWM is discussed and the expected improvements resulting from the amendment are presented. In Chapter Four Richards' equation is described, together with boundary conditions suitable for a natural catchment. Attention is focused on a one-dimensional, vertical solution to flow in a "column" of soil as providing the most efficient means to test the thesis expressed in Section 1.2D, and the details of its numerical solution are given.

(b) Chapters 5-8: Amending the Stanford Watershed Model

The numerical solution and inclusion into the SWM of the one-dimensional form of Richards' equation are described in Chapter Five. Chapter Six lists the catchments from which data were obtained to test the Amended Model (AM), the choice of time step for the simulations and the method of evaluating the model parameters. The results of testing the AM against the basic SWM are presented and discussed. Chapter Seven then uses the performance of the AM, plus the published information on more general solutions to Richards' equation, to predict the usefulness of continuing this emphasis on the subsurface zone. Finally, Chapter Eight presents the conclusion and suggests certain areas for further work.

CHAPTER TWO

DIGITAL CATCHMENT MODELS

2.1 What is a Digital Catchment Model?

A. A Catchment

From a given point on a river the two lines of steepest upward slope will eventually join to enclose an area known as the catchment of that point. Because of the traditional emphasis on surface flow as a storm response mechanism, riverflow at this point is assumed to be derived entirely from precipitation occurring on the catchment. Engineering interest in riverflow therefore extends to a consideration of the precipitation on the catchment, and the way in which the catchment subtracts from and delays the precipitation on its path to the river.

B. A Model

One system may be modelled by another if an analogy exists between the two systems, enabling the behaviour of one system (the model) to be used to predict the behaviour of the other (the prototype). The model usually has certain aspects, such as size or speed of operation, which make measurements on it easier to obtain than on the prototype. Models may take three forms:

(a) Physical Models, which differ primarily in size from the prototype.

(b) Analogue Models, which are mechanical, electrical or other devices constructed to have features corresponding to those of the prototype.

(c) Mathematical Models, which represent the behaviour of the prototype by mathematical equations or functions. For example Darcy's equation, $v = - K\,i$, represents the linear dependence of velocity v in a saturated porous medium with hydraulic conductivity K on the potential gradient i. The operation of the prototype is modelled by solving the equation(s).

The term "system" is frequently used in the remainder of this thesis to refer to the catchment area, together with the earth below and the vegetation above. The terms "input" and "output" refer to the flow of water into (by precipitation) and out of (by riverflow) the system. The task of a mathematical model, then, is to find a function which duplicates the input-output behaviour of the system. Section 2.2 discusses a form in which the input, output and the system parameters may be expressed.

C. Digital Models

A mathematical model becomes digital when numerical values are given to the terms in the equations or functions. For example Darcy's equation $v = - K\,i$ is not digital as such, because we do not need values for the terms in order to draw inferences about its behaviour. But if we specify a value for the hydraulic conductivity K, which "fits" the general model to a particular porous medium, and if we supply a series of (digital) values of potential gradient i, then a corresponding series of (digital) values of velocity v can be calculated. Solving the equation for the velocity v has modelled the behaviour of the prototype.

D. Digital Catchment Models

Following the above definitions, a Digital Catchment

Model can be described as a mathematical function whose input-output behaviour is analogous to the precipitation-riverflow behaviour of a catchment. It is used by supplying values to the coefficients in the various equations which comprise the function and by supplying numerical values of the precipitation. The function can then operate on these numbers to produce numerical values of riverflow.

2.2 Input and Output

A. Discretisation

Precipitation and riverflow are continuous functions of time. In order to input these to a digital model they must be approximated by finite arrays of values, usually rates or volumes at regular time intervals. The choice of time interval depends on the time-scale of the fluctuations it is desired to simulate; prediction of peak flows requires time intervals of minutes or hours, while long-term volumes can be predicted using intervals of days or months. The time interval needed for peak prediction also depends on the response speed of the catchment, since the output of a slowly-responding catchment will not reflect small-period fluctuations in the input. Once a time interval has been chosen, the catchment instrumentation must be able to resolve events to this accuracy.

The time intervals required to describe a very short-lived peak will be smaller than those necessary for periods when conditions are changing less rapidly. The proposal of Langham[6] to represent the time functions of precipitation and riverflow by arrays of <u>variable time</u> intervals required to accumulate a <u>constant volume</u> of precipitation or riverflow would save both data storage and model calculation time, at the expense of computational simplicity. However Ibbitt[7] recommended that standard data sets for the comparative testing of catchment models should be at fixed time intervals. A possible compromise is to store data at fixed time intervals, but to make provision for combining time steps at the calculation stage in periods of relative inactivity.

B. Precipitation

Precipitation is commonly assumed to be the only water input to the catchment. Thus subsurface flow into the system is ignored. Precipitation is a continuous function of distance as well as time, but since it can be measured at only a few points on a catchment it is necessarily discretised with respect to area. Usually a "catchment average" precipitation is calculated for each time interval, it being assumed that this fell over the entire area.

Precipitation is almost always measured by devices which record accumulated volume at a point. So the precipitation is usually represented by an array of numbers describing the volumes of catchment average precipitation in successive equal time intervals.

C. Flow

Model output may consist of flow rates or flow volumes at regular time intervals. To evaluate the success of a simulation this output must be compared with the recorded riverflow, so the form in which the latter is available will often determine the form of the model output.

Riverflow is measured by a river stage recorder, being translated to flow via a stage-discharge relation applied at successive time intervals. Thus the primary form for riverflow data is a series of rates; these can be integrated to obtain volumes.

D. Evaporation

Not all the precipitation which falls on a catchment becomes riverflow. Most of that which does not is evaporated from the land surface or transpired by plants. The term evapotranspiration is used to describe the combination of these two processes.

If a model calculates evapotranspiration in the process of calculating flows it requires some estimate of the available energy; a convenient form is the "potential evapotranspiration". This is the evapotranspiration rate which occurs in the absence of any restrictions on the supply of moisture. This information may be obtained by observing the rate of change of water level in an open container of water (an evaporimeter), or by observing other meteorological variables (such as wind speed or temperature) which correlate with evaporimeter readings.

E. Parameter Values

Every mathematical model contains one or more equations or other mathematical functions, and these contain coefficients which, given numerical values, turn a general model into a model representing a particular system. For example the hydraulic conductivity in Darcy's equation fits the equation to a particular porous medium. These coefficients are termed the model parameters.

Generally there is more than one parameter, and since there is usually little information about the system between the input and the output these cannot be evaluated independently. Consequently parameter values must be estimated using knowledge of the physical processes which make up the input-output function, or they must be evaluated by trial-and-error fitting of the simulated output to the recorded output.

2.3 Classification

A. General

Models which represent a catchment in the deterministic sense may be classified as shown in Figure 2-1. The divisions are discussed in general in Part A of this section, while Parts B to E describe each class of model in more detail.

(a) Conceptual Models

A form for the input-output relationship of the system is postulated using knowledge (or assumptions or conceptions) of the physical nature of the processes involved. The form adopted is influenced by limitations in the model-builder's knowledge, by restrictions of data availability and by computational restraints. Although the form can be expressed in equations and other mathematical functions which describe real processes, many of the physical properties (parameters) cannot be easily estimated for a particular catchment because they represent average conditions for a catchment which is undoubtedly very heterogeneous.

(b) Analytic Models

Because of the difficulties of constructing a conceptual model, an alternative approach has been widely used. The mathematical form postulated for the input-output

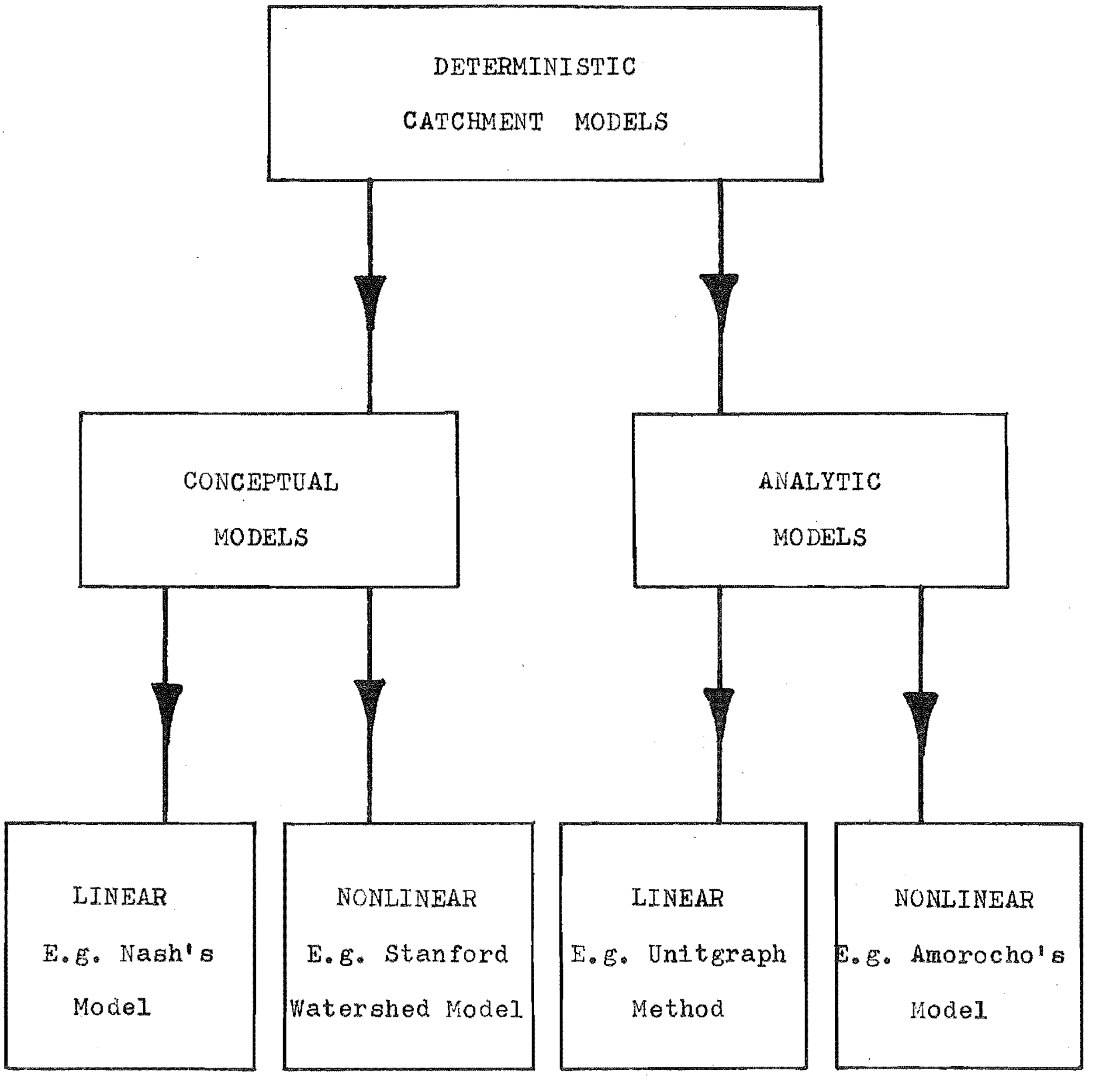


FIGURE 2-1: CLASSIFICATION OF CATCHMENT MODELS.

relationship is made general enough to be fitted to a wide variety of real data, without making any assumptions about the physical processes involved. Solutions for this mathematical form are found from recorded sets of input and output data. This is especially convenient if the assumption of linearity is made.

(c) Linearity

Both conceptual and analytic models may be linear or nonlinear. Linearity implies that each increment of input produces the same time pattern of output, with ordinates proportional to the magnitude of the input increment. The output resulting from any general input may be found by superposition once the output resulting from a unit input is known; this unit response, which typifies the input-output relation, can be found using recorded input-output data.

However the action of a catchment in translating precipitation into riverflow is not linear, and various assumptions such as subtracting "losses" and "baseflow" are commonly made to fit the data to the theory. Thus these methods must be evaluated together with methods for simulating loss and baseflow, and not in isolation.

B. Linear Analytic Models

This class is typified by the Unitgraph Method introduced by Sherman(8) in 1932. He postulated that after making adjustments to the input and output data to equate their volumes, the relation between them is linear. The system could then be described completely by the output resulting from an input of unit volume, which he called the Unitgraph. To calculate the response to another input, the unitgraph could be used without making any assumptions about the catchment processes except that they are linear and time-invariant.

The response to any general input is found by summation, which can be written:

$$q(t) = \sum_{\tau=0}^{\infty} P(t-\tau)\ U(\tau) \qquad (2\text{-}1)$$

That is, the output q at time t is contributed to by input increments from a time τ previously $P(t-\tau)$, multiplied by the unitgraph ordinate $U(\tau)$. This is the discrete form of the summation; as the time interval at which the input is discretised tends to zero the summation becomes an integral known as the Convolution Integral:

$$q(t) = \int_{\tau=0}^{\infty} p(t-\tau)\ U(\tau)\ d\tau \qquad (2\text{-}2)$$

However, before the unitgraph can be found from a given storm, the output due solely to that storm must be separated from the slowly-receding output from all past storms. Various methods with little basis other than convenience are used to subtract this "baseflow", leaving what is known as "direct runoff". Then the input volume must be adjusted to agree with the volume of direct runoff, since the linear theory makes no provision for any losses. This is usually done by subtracting a constant "loss rate" from the input, leaving the "rainfall excess". Hence the unitgraph theory is concerned only with translating rainfall excess into direct runoff. To predict the riverflow resulting from any given precipitation other models must be called upon to simulate the remainder of the system.

Although widely used because of its simplicity, the unitgraph method does not model the complete input-output relation, its linear and time-invariant assumptions are questionable and the model is not capable of development resulting from improved knowledge of the hydrologic cycle.

C. Nonlinear Analytic Models

The process by which a linear function transforms an input into an output by the convolution operation may be generalised to include higher order processes which are not in general linear[9]. This removes the need to make arbitrary loss or baseflow separations, and the complete rainfall-riverflow process may be simulated. Solution for the input-output function is more complex than for the linear case, and investigators have used series expansion approximations[10] and multiple regression approaches[11] to solve for the function.

The method stil requires the time-invariance assumption, is complex to use and cannot include knowledge of the hydrologic cycle. However it can model the complete input-output relation.

D. Linear Conceptual Models

Investigators seeking to predict the unitgraph for a catchment without records first considered the catchment as a storage of water. A linear model to represent storage is the linear reservoir whose output is proportional to its content; this has both a damping and a delaying effect on an inflow in a manner similar to that of a catchment on a rainfall (Figure 2-2).

Zoch[12] first proposed the linear reservoir to account

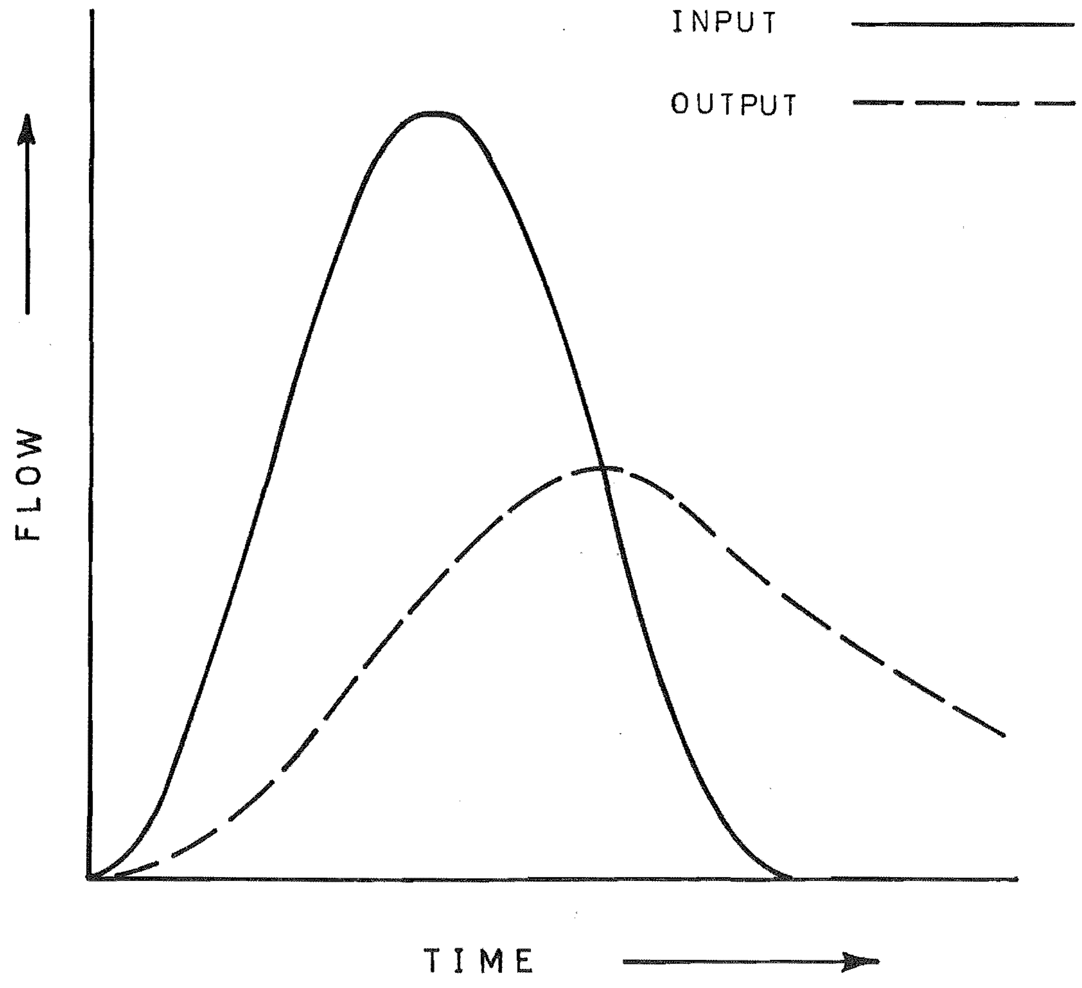


FIGURE 2-2: THE ACTION OF A LINEAR RESERVOIR

for storage in the catchment. Clark[13] used the Time-Area-Contributing-Diagram (TACD) to represent the time distribution of runoff resulting from a unit depth of rainfall excess over the whole catchment and modified this to allow for storage by routing it through a linear reservoir. The resulting unitgraph was therefore a one-parameter (the reservoir proportionality constant) model of the rainfall excess-direct runoff process.

A similar approach was made by Nash[14]. Assuming the storage effect of the catchment was more important than the travel-time effect, he ignored the TACD and passed a unit pulse of rainfall excess through a series of identical reservoirs, the output from one being the input to the next. He obtained an analytic expression for the final output in terms of the reservoir constant and the number of reservoirs; his unitgraph was therefore a two-parameter model.

Dooge[15] generalised all previous methods by synthesising the unitgraph from a series of linear reservoirs (concentrated storage) and linear channels (pure translation, or distributed storage). Although computational reasons restricted solution to the case of equal reservoirs distributed according to the TACD of the catchment, his model of the unitgraph was the most versatile. However, linear conceptual models suffer from all the disadvantages of linear analytic models discussed previously.

E. Nonlinear Conceptual Models

Removal of the restriction of linearity enables the model-builder to use as much information about the hydrologic cycle as he can get, and to simulate if he wishes the complete precipitation-riverflow process. Because there are widely-varying types of models in this category, they are discussed in several groups.

(a) Nonlinear Reservoirs

Laurenson[16] used a reservoir model but removed the linearity constraint. He divided the catchment into subareas of equal travel-time to the outlet, based on a parameter given by distance divided by the square root of slope. A reservoir in each subarea represented the storage effect of that subarea, both for rainfall excess within the subarea and also for output from subareas more distant from the outlet. The model recognised areal inhomogeneity of runoff production, since rainfall excess from the most distant subarea had to pass through the maximum number of reservoirs, while excess from the closest subarea was delayed by only one reservoir. Nonlinearity was included by allowing the reservoir constant to vary with the flow rate, but the model simulated only the rainfall excess-direct runoff part of the system.

(b) Container Models

Another group of models employs a series of nonlinear reservoirs representing storage in the various assumed components of the hydrologic cycle rather than in different areas of the catchment. For example the Stanford Watershed Model[17] (SWM) has storages to represent interception, soil moisture, surface water, interflow and groundwater. During each time step of a simulation, the current precipitation volume is added to one of the reservoirs, and transfers both between reservoirs and into the river are made. The transfer operations embody the current knowledge of the physical processes involved; they thus depend on both the catchment properties (as described by the model parameters) and on the current distribution of water in the catchment (as described by the current states of the model reservoirs).

However, with the number of storages usually involved there is a large number of transfer equations, each including at least one coefficient or parameter. Because the storages and transfers represent average catchment conditions many of the parameter values are difficult to estimate from knowledge of the catchment alone. The parameters therefore have to be evaluated by a trial-and-error adjustment which fits the model output resulting from a given recorded precipitation as closely as possible to the corresponding recorded riverflows. In spite of this difficulty, this type of modelling has been very popular because of the potential for improvement as basic hydrologic research provides more information about each of the model components. Other significant models of this type are those of Boughton[18] and Porter and McMahon[19]; considerable work on the parameter evaluation problem has been done by O'Donnell and his co-workers[20].

(c) Overland Flow Models

The equations of two-dimensional flow with lateral inflow can be applied to a river channel or to an impervious catchment such as a parking lot. The traditional emphasis on surface flow as the major storm response mechanism prompted the application of these equations as a model for the rainfall excess-direct runoff process. In this method the catchment is represented by a sloping plane, the rainfall excess represents the lateral inflow and the outflow from the bottom of the plane represents direct runoff. The catchment parameters are the slope of the plane and its roughness characteristics.

Henderson and Wooding[21] employed an approximate form of these Overland Flow equations, known as the Kinematic Wave approximation, to create such a model. Later work by Woolhiser and Ligget[22] showed by comparison with the full equation solution that the Kinematic Wave approximation was satisfactory for flow over natural surfaces. In addition Wooding[23] used the Kinematic Wave approximation to solve for flow from a Vee-shaped catchment consisting of two planes (Figure 2-3). Rainfall minus infiltration was the lateral inflow to the overland flow on the sides of the Vee, while the outflow from the bottom of the planes was the lateral inflow to the channel flow in the bottom of the Vee. Wooding used this to model several real catchments, although he first had to subtract loss and baseflow by traditional methods to obtain his data.

(d) Subsurface Flow Models

Mathematical description of flow through porous media has long been possible; even the more complex case of unsaturated flow has been capable of solution following work in the irrigation and petroleum fields. Important contributions were made by Richards[5], who first wrote the equation of motion with moisture content-dependent coefficients and after whom this form of the equation is named; Klute[24], who first solved Richards' equation numerically, and Philip[25]. Philip's analytic solution for the infiltration resulting from a step increase in moisture content is a series which for small values of time can be written as:

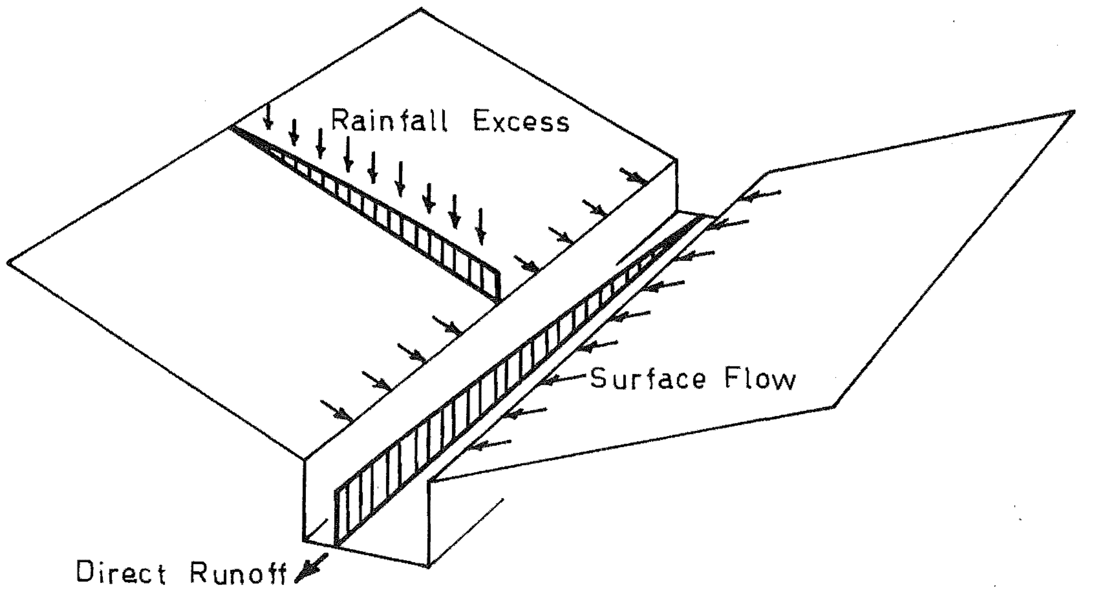


FIGURE 2-3: WOODING'S CATCHMENT MODEL

$$i(t) = \frac{1}{2} S t^{-\frac{1}{2}} + A \qquad (2\text{-}3)$$

where i(t) is the infiltration rate at time t after the increase, and S and A are constants depending on the medium and the initial and final moisture contents. This equation has been used to calculate infiltration in some conceptual models, but infiltration resulting from a more general moisture supply requires a numerical solution.

Numerical solutions of Richards' equation written in the vertical direction only have been used to calculate infiltration in several conceptual models. Smith and Woolhiser(26) employed a number of parallel solutions to evaluate losses from the lateral inflow to a Kinematic Wave solution in their model. Harley, Perkins and Eagleson(27) described a constant-diffusivity solution to be used for infiltration or exfiltration in a similar model but it was not actually implemented. In both cases the infiltrated water played no further part in the simulation and was considered a loss.

Freeze(28) presented a one-dimensional, vertical solution to Richards' equation in which the unsaturated zone was solved in conjunction with a dynamic saturated zone; the boundary between them was allowed to vary in response to applied inputs and outputs. He later generalised this to a three-dimensional solution(29), and solved for a block of porous medium typical of a catchment subject to various applied rainfalls. Such a solution would seem to be appropriate to represent a catchment whose response did not occur via surface flow, or to represent the subsurface zone in a general catchment model.

CHAPTER THREE

TOWARDS A BETTER MODEL

3.1 Model Uses

This chapter discusses the ways in which catchment model performance might be improved in general, before choosing the specific way in which to test the postulation that an improved subsurface description will improve model performance.

The choice of a model, or of any improvement envisaged for a model, depends on the purpose for which it is to be used. Given that peak flows rather than long-term volumes are of interest, the ways in which a model can be used are:

(a) Short-term Forecasting

Weather forecasts enable estimates of rainfall to be made in advance, for periods up to a few days. A catchment model can calculate the flow resulting from the expected rain; such information is especially useful in time of anticipated flooding (Figure 3-1).

(b) Long-term Prediction

Engineering design of structures associated with waterways requires knowledge of the "design flood", a flow whose probability of exceedance is acceptably small. Since floods of this size are by definition quite rare, the determination of this flow requires as long a flow record as possible. However, flows have been recorded for shorter periods than rainfalls. Thus if a model of sufficient accuracy is available, improved estimates of future flows of specified frequencies can be found by (i) estimating

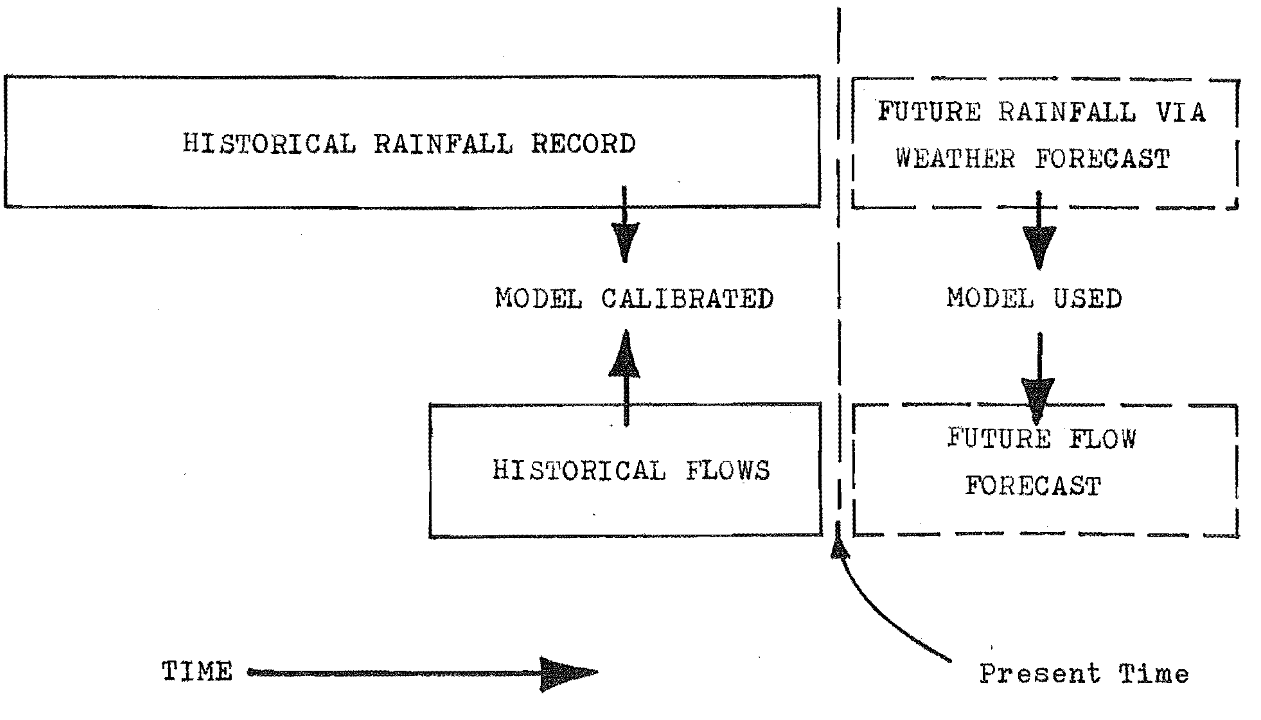


FIGURE 3-1: SHORT-TERM FLOW FORECASTING

possible future rainfall events and converting these to flows with the model, or (ii) filling in the gaps in the historic flow record using the model on the historic rainfalls. Ibbitt[7] calls these "construction" and "reconstruction" respectively; the two approaches are illustrated in Figure 3-2.

(c) Understanding of the Hydrologic Cycle

The successful simulation by a physically-based model of the precipitation-riverflow process serves to confirm, though not to prove, that the concept of the hydrologic cycle held by the model-builder is a good representation of the actual process. An important consequence of this is that the model may be used to examine the effects of any proposed changes in the catchment, merely by changing the model parameters.

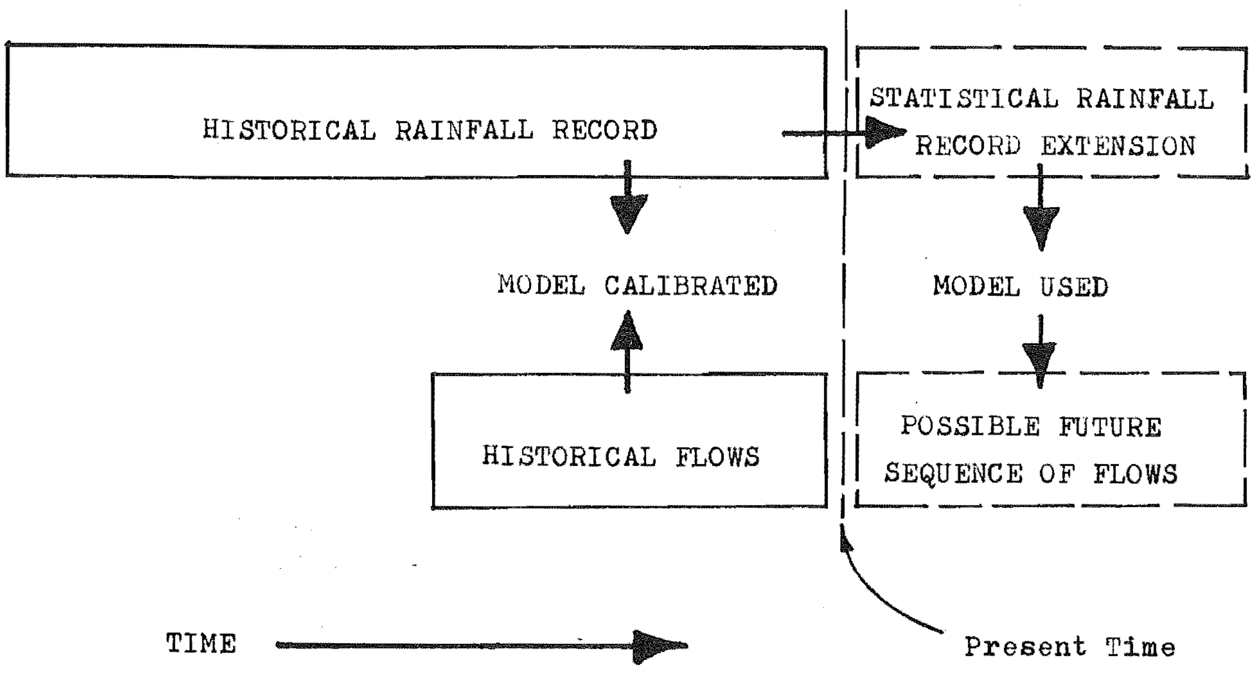
3.2 Which Model Type?

A versatile model must be capable of being used for all three purposes above. With this in mind, the type of model can now be chosen.

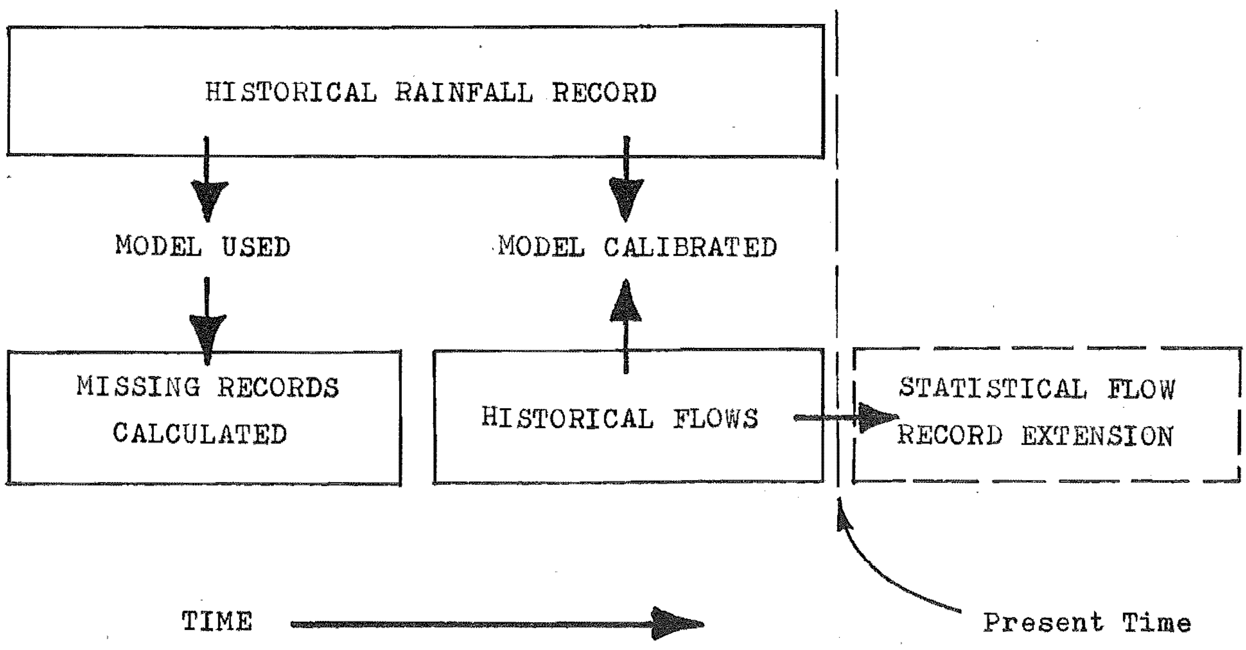
A. Linear or Nonlinear?

Linear methods invariably simulate only part of the system, requiring additional nonlinear steps to represent loss and baseflow. The introduction of nonlinear steps to a linear model makes the total model nonlinear; this reduces the major advantage of linearity, that of simplicity.

It is the opinion of the writer that for the above purposes the best model would be nonlinear.



(a) Flow Prediction Via Rainfall Record Extension ("Construction")



(b) Flow Prediction by Calculating Missing Records ("Reconstruction")

FIGURE 3-2: LONG-TERM FLOW PREDICTION

B. Analytic or Conceptual?

Given the limited capability of measuring and describing all the relevant properties of a catchment and the present imperfect knowledge of the catchment processes, an analytic model would seem attractive for purposes (a) and (b) in Section 3.1. But purpose (c) requires that a change in the catchment properties be able to be related to a change in the model; this cannot be done for an analytic model.

Hence a conceptual model will be required to achieve the three purposes in Section 3.1, notwithstanding that analytic models may at present perform better in some circumstances.

3.3 Which Model?

Since it is unlikely that any completely novel approach to modelling will be found, an existing model of acceptable performance is required into which may be inserted the intended improvements. In addition, this model will serve as a yardstick of performance against which the amended model may be judged.

The Stanford Watershed Model (SWM) was chosen as a "base" model for the following reasons:

(a) The SWM is a nonlinear, conceptual model which simulates the complete system.

(b) The SWM is capable of operating at the small time intervals required for peak simulation.

(c) The SWM is physically-based, so that the structure is capable of accepting revised models of component processes as improved knowledge allows.

(d) Performance of the SWM is good[17,30,31]. Although the many parameters are hard to evaluate, the model has the flexibility to fit given records well and to predict other records satisfactorily.

3.4 What is Wrong?

The performance of a catchment model is determined not only by the nature of the function which translates input into output but also by how well the input, output and the catchment itself are represented. In order that these latter features do not obscure a possible improvement in the function specification, any comparison must be made under controlled conditions. The way in which each aspect affects the performance is examined in this section, using the SWM as an example.

A. Input-output Description

Rainfall varies with position over a catchment. Input to the SWM however is in the form of average volumes of rainfall on the catchment. The error involved in this approximation depends on the density of the raingauge network, the variation in rainfall over the catchment and the inhomogeneity of the catchment properties; it is at a minimum on a small catchment. The restriction in Chapter One of this study to hydrologically small catchments will minimise this problem.

Both rainfall and riverflow vary continuously with time. Rainfall and riverflow are described in the SWM as volumes or rates at regular time intervals, so the problem is posed as, "what time interval is required to represent these functions to an acceptable accuracy?" An acceptable accuracy for this study is defined in Section 6.3, together with a method for determining the corresponding time intervals.

B. Catchment Description

The characteristics which affect the response of a catchment to rainfall also vary with position over the catchment, but the "integrating" action of a catchment in focusing distributed rainfall into flow at a point in a river encourages the view that effective average values of catchment properties may be found. Most of the components in the SWM employ average catchment parameters. As for input description the error involved here will be minimised if the catchments are small.

Temporal variations in the catchment properties may be short- or long-term ones. Short-term changes arise mainly from changes in the moisture status of the catchment, and this is modelled by the specification of the equations comprising the SWM. Long-term variations include urbanisation and afforestation; errors from this source will be avoided if the data used covers only a short period of time. However if such a change occurs suddenly, for example the removal of forest by burning or milling, comparison of before and after simulations may help to evaluate some of the model parameters[32].

C. Model Structure

The function which represents the rainfall-riverflow process should follow as closely as is practicable the physical laws governing the flow of water over and under the land surface. Other premises invite the liklihood of the function not being applicable outside the range of conditions for which it was derived or calibrated. The equations governing flow over natural surfaces and through porous media are known, and can be solved numerically for certain configurations; so in principle the catchment could be represented by solving the equations for a block of soil the same shape as the catchment (Figure 3-3). Freeze and Harlan[33] have described in detail this long-term goal.

How closely this ideal can be approached depends on the computational resources available; even with the rapid growth in the capability of digital computers in the last decade such a complete solution is still barely possible. So the SWM uses the equations of motion only to describe surface flow, and uses interconnected reservoirs to describe storage below the surface. Transfer of water between the reservoirs (see Figure 3-4) is controlled by equations which involve coefficients (the model parameters, labelled a or x in Figure 3-4) and by the current amounts of water in the reservoirs. These equations represent the following processes:

Interception
Infiltration
Interflow
Percolation from Topsoil to Subsoil
Groundwater Flow
Evaporation
Riverflow

Experience gained by the writer using the SWM on several New Zealand catchments[31] suggested several areas in which the model might benefit from modification:

(a) Physical Reality

While the rules for transferring water between the

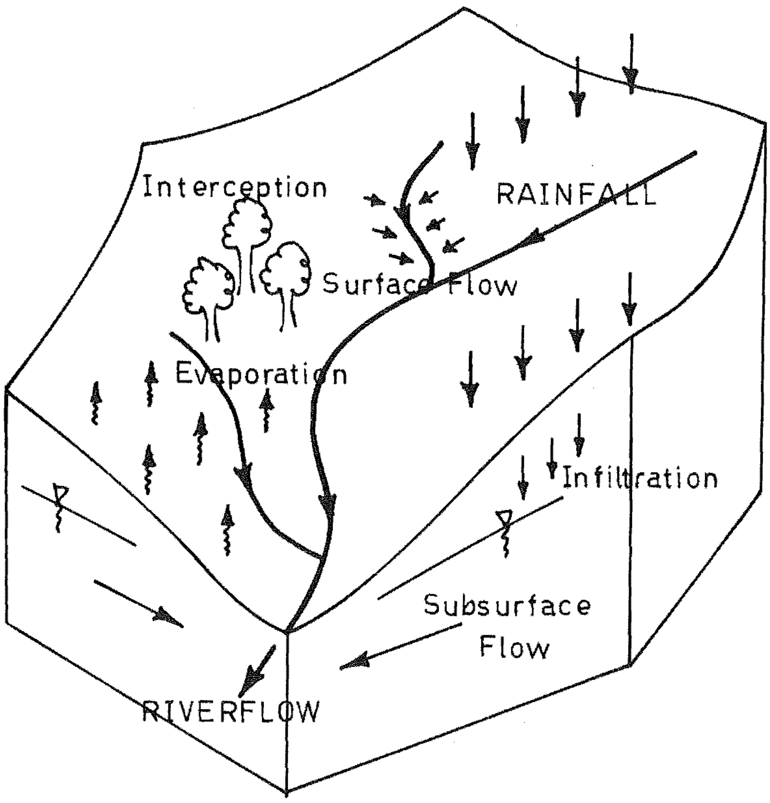


FIGURE 3-3: IDEALISED VIEW OF A CATCHMENT

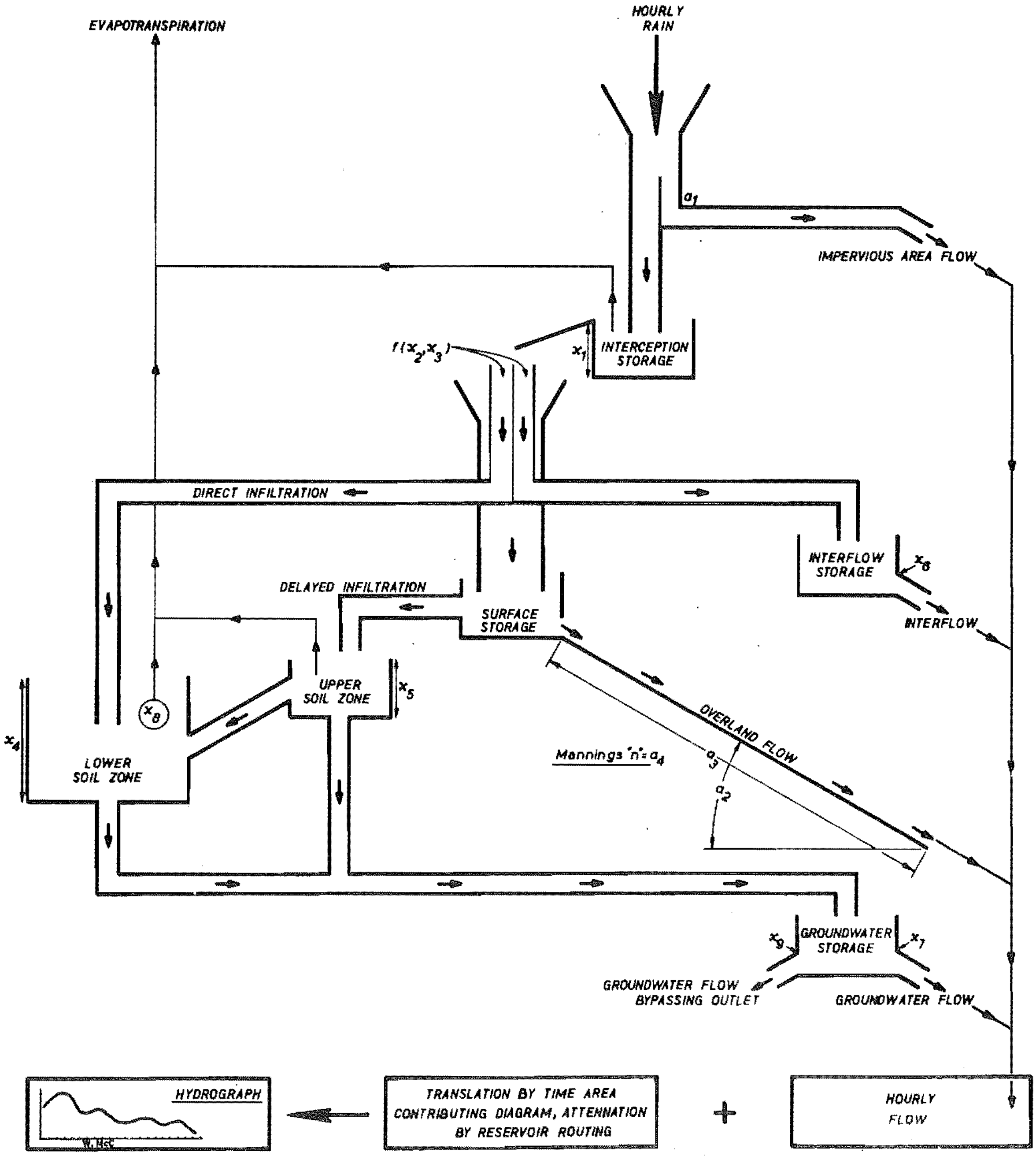


FIGURE 3-4: THE STANFORD WATERSHED MODEL

storages of the model are based on sound reasoning, they frequently retreat into empiricism.

(b) Partial Area Runoff

Although the infiltration equations allow for areal inhomogeneity, the surface water thus calculated becomes input to the surface flow equation solution which assumes the whole catchment to be taking part. As pointed out by Kirkby and Chorley[34], catchment-wide surface flow represents only one end-point of a range of response mechanisms, the other end of which is represented by only the small area adjacent to the river contributing promptly.

(c) Component Processes

The model makes artificial distinctions between various forms of subsurface response. In reality there exists a continuum of ways in which a drop of rainfall may travel to the catchment outlet.

(d) Areal Variation

Apart from allowances made in the infiltration and evaporation components the catchment is considered areally homogeneous. Even in these two processes the variation in the catchment property must conform to a mathematically convenient form.

(e) Parameter Estimation

Many of the large number of parameters (there were 13 in the version implemented by the writer) were difficult to estimate because of lack of relation to a measurable quantity. Evaluation by fitting was also difficult because many parameters affected the simulation in a similar manner.

3.5 How to Improve?

The aim of this study is to determine whether Richards' equation will describe the flow in the subsurface zone of a catchment more successfully than methods used in existing models. Having chosen an existing model as a basis for improvement and comparison this aim can now be directed into a course of action.

Previous applications of Richards' equation in a hydrologic context[26,27] have represented only a loss from the rainfall input to a surface-predominant model, or have been concerned[28,29] only with demonstrating the features of a solution to the equation using artificial data. It remains to be shown whether a Richards' equation solution to a block of porous medium representing a catchment can simulate both infiltration and subsurface flow in a model used on real data. Such a model might be expected to offer, at the expense of computational simplicity, the following advantages over the SWM:

(a) Physical Reality

Richards' equation is a more appropriate description of the movement of water in the soil than the empirical relations of the SWM.

(b) Partial Area Runoff

The changing area of the catchment over which rainfall exceeds infiltration may be able to be predicted by the equation solution. It is expected that by varying the soil properties the surface-responding, subsurface-responding, and all intermediate combinations of catchment types could be modelled.

(c) Component Processes

Use of Richards' equation to solve for the whole subsurface zone removes the need to make arbitrary distinctions between various subsurface flow mechanisms.

(d) Areal Variation

The equation, depending on whether it is solved in one, two or three dimensions, can allow for varying degrees of inhomogeneity in the soil properties.

(e) Parameter Estimation

Although there might be a larger number of model parameters, especially where inhomogeneity exists, parameter estimation would be easier. This is because greater faithfulness to the real system will facilitate the measurement of more parameters in the field, thereby reducing the number to be found by trial-and-error.

The general aim of this study as stated in Section 1.2D may now be expressed in more definite terms: to determine whether the replacement of the subsurface components of the SWM by a solution to Richards' equation will improve the model's ability to simulate peak flows on hydrologically small catchments.

CHAPTER FOUR

FLOW IN UNSATURATED POROUS MEDIA

4.1 Background

This chapter describes the generalisation of Darcy's equation for flow in saturated porous media to Richards' equation for unsaturated porous media. Suitable boundary conditions for the catchment situation are presented and the various configurations of the solution which might be used in a catchment model are discussed.

Darcy's equation in vector form is:

$$\bar{v} = -K \nabla h \tag{4-1}$$

where $\bar{v}$ is the velocity vector, whose magnitude is the discharge divided by the gross area of flow,

K is a characteristic of both the porous medium and the fluid, known as the hydraulic conductivity (a tensor),

h is the hydraulic head, with

$$h = p/\gamma + z$$

p is the gauge pressure

γ is the specific weight of the fluid, and

z is the elevation head

In an unsaturated porous medium the pressure is replaced by the soil suction, which is a measure of the attraction exerted for the fluid by molecular and surface tension forces in a dry medium. This attraction increases from zero at saturation to many atmospheres as the medium dries out (see Figure 4-1). The origin of these forces is described by White et al.[35], and the complementary nature of soil suction and pressure is illustrated in Figure 4-2. Since gradients of suction tend to move fluid from areas of low to areas of high suction, in direct contrast to gradients of pressure, soil suction may be regarded as a negative pressure enabling hydraulic head to be defined as for the saturated case.

The relation between the hydraulic head gradient and the velocity in an unsaturated porous medium may be expressed in a manner similar to Darcy's equation, provided the hydraulic conductivity is allowed to vary with the moisture content. The value of K becomes smaller as the medium dries out because the fraction of the gross area participating in the flow decreases, and the flow paths become more tortuous (see Figure 4-3).

4.2 Derivation

According to the arguments of the last section we have Darcy's equation applicable to an unsaturated porous medium:

$$\bar{v} = -K(\theta)\,\nabla h \tag{4-2}$$

where K the hydraulic conductivity is now a function of moisture content,

h the hydraulic head is now given by

$$h = \psi(\theta) + z$$

ψ is written for p/γ , recognising that it may be

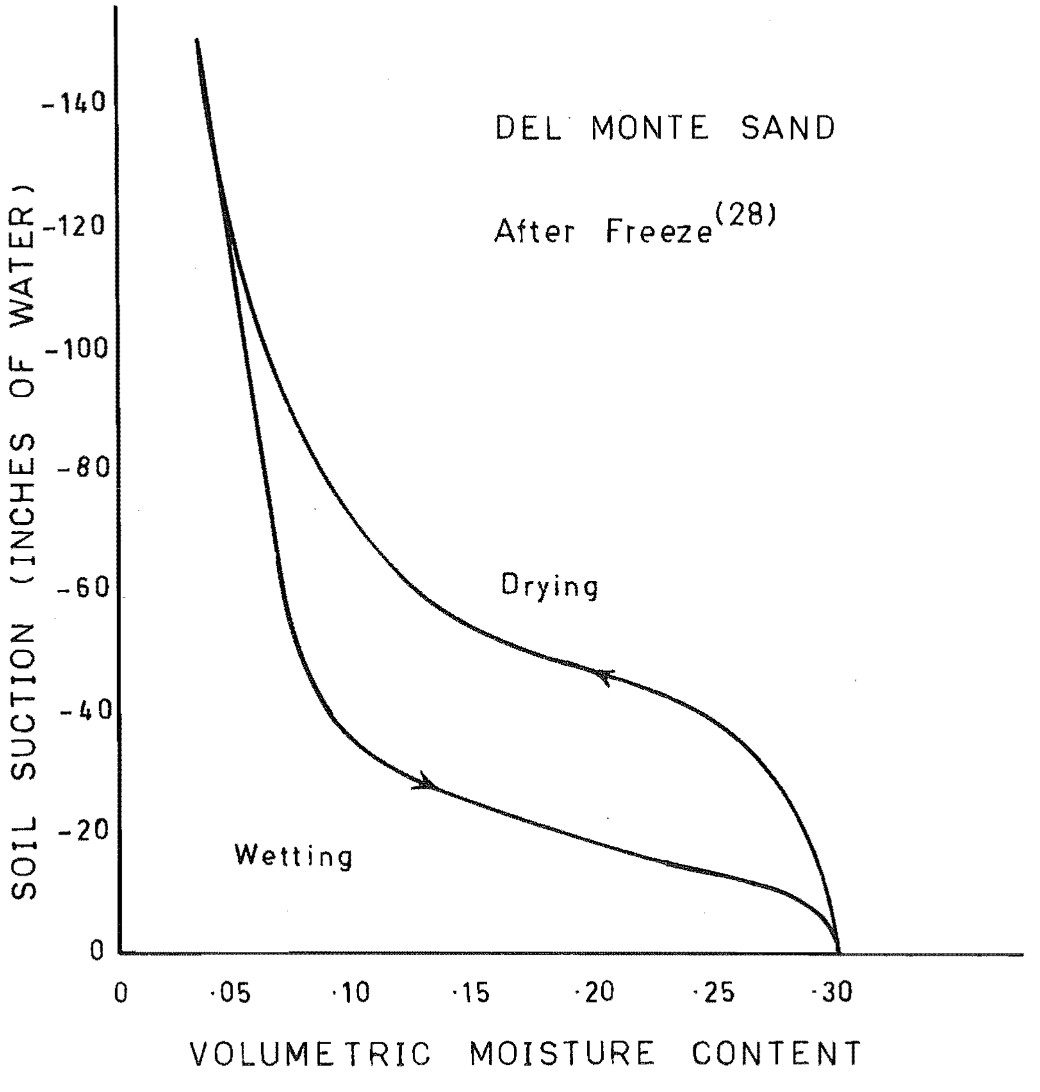
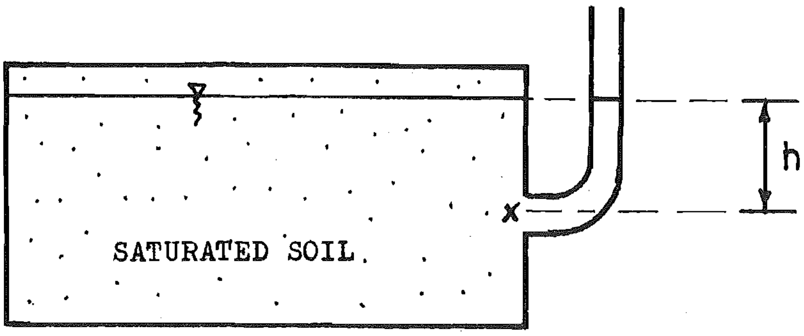
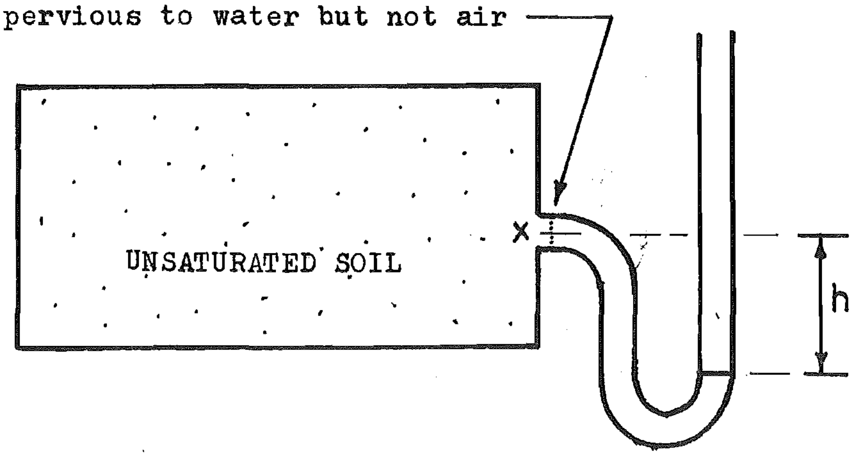


FIGURE 4-1: VARIATION OF SOIL SUCTION WITH MOISTURE CONTENT



(a) Saturated Soil: In the absence of the manometer, a POSITIVE pressure of γh would have to be applied at X to prevent flow (which would be OUTFLOW).

Membrane pervious to water but not air



(b) Unsaturated Soil: In the absence of the manometer, a NEGATIVE pressure of γh would have to be applied at X to prevent flow (which would be INFLOW).

FIGURE 4-2: THE COMPLEMENTARY NATURE OF SOIL SUCTION AND PRESSURE

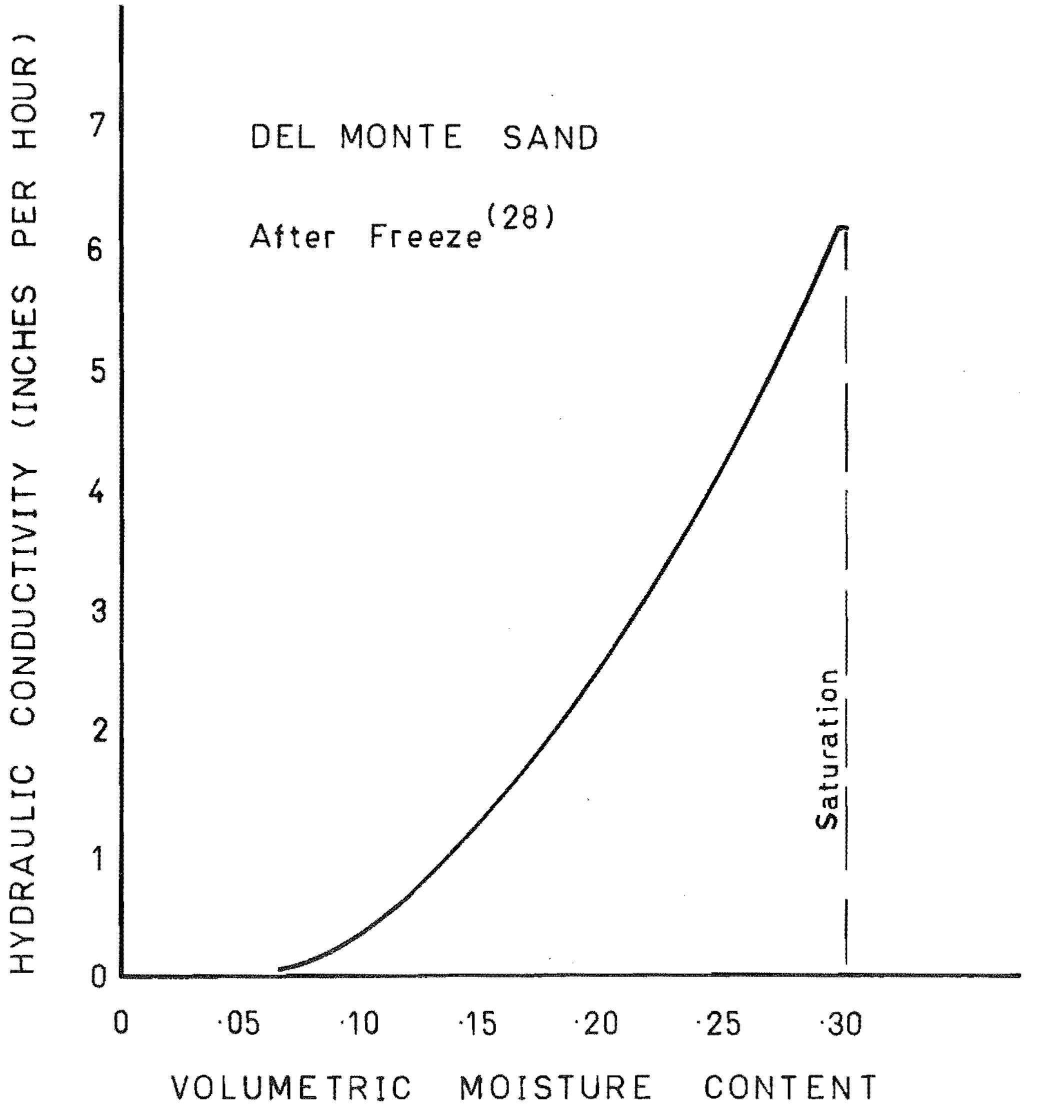


FIGURE 4-3: VARIATION OF HYDRAULIC CONDUCTIVITY WITH MOISTURE CONTENT

positive (pressure head) or negative (soil suction), and that it is a function of the volumetric moisture content θ .

Assuming the fluid is incompressible, the continuity equation is:

$$\nabla \cdot \bar{v} = -\frac{\partial \theta}{\partial t} \tag{4-3}$$

where t is time. Combination of equations 4-2 and 4-3 yields:

$$\frac{\partial \theta}{\partial t} = \nabla \cdot \left\{ K \nabla (\psi + z) \right\} \tag{4-4}$$

which is known as Richards' equation. If we now define the Specific Moisture Capacity $C(\psi)$ as:

$$C(\psi) = \frac{d\theta}{d\psi} \tag{4-5}$$

we can express equation 4-4 as:

$$C \frac{\partial \psi}{\partial t} = \nabla \cdot \left\{ K \nabla (\psi + z) \right\} \tag{4-6}$$

Alternatively, equation 4-4 may be expressed in a θ-based, rather than a ψ-based, form by introducing the Diffusivity $D(\theta)$, defined by:

$$D(\theta) = K \frac{d\psi}{d\theta} \tag{4-7}$$

Although the θ-based form is more convenient for analytic solution it is applicable only to unsaturated media. Equation 4-6 is based on ψ, which retains meaning in both saturated and unsaturated porous media. It must however be solved numerically. Digital modelling of a catchment is essentially a step-by-step numerical process, so a numerical solution to an equation is easily accommodated. In addition the soil in a catchment will become saturated at times, precluding the use of the θ-based form of Richards' equation. Equation 4-6 is therefore the most suitable form of Richards' equation to use in a digital catchment model.

The assumptions made in deriving equation 4-6 are now collected:

(a) Darcy's equation, as defined for an unsaturated porous medium, is valid. The wide range of conditions in which Darcy's equation is successfully used supports this assumption.

(b) Water movement resulting from temperature, osmotic and chemical concentration gradients is negligible. This is thought to be reasonable in a hydrologic context.

(c) The hydraulic conductivity and moisture content are unique functions of soil suction. Hysteresis does exist between wetting and drying in these relations, so this assumption is satisfied only when the medium is monotonically wetting or drying. The values of conductivity and moisture content at a given suction can vary by a factor of two or more between wetting and drying, but in the storm situation which is of most concern to the catchment modeller the soil would be monotonically wetting.

(d) The hydraulic conductivity and moisture content are time-invariant functions of soil suction. This implies that changes in the structure of the medium, for example by swelling or compaction, are small. On most catchments the effect of these factors on the hydrologic properties of soils will be small compared to the effect of changes in the moisture status of the soil.

(e) Water is incompressible. This is the usual assumption in both saturated and unsaturated flow.

(f) The medium offers no resistance to the flow of the non-wetting phase, usually air. Sufficient inhomogeneity to allow the escape of entrapped air will always be present in a natural catchment.

Solution of equation 4-6 may now be attempted provided that the relations between hydraulic conductivity, moisture content, specific moisture capacity and soil suction are known, and that appropriate initial and boundary conditions are specified. Initial conditions may be any realistic distribution of soil suction (or moisture content); boundary conditions are discussed in the next section.

4.3 Boundary Conditions

Boundary conditions may be specified either by values of Ψ or by its gradients. An especially useful form of the latter enables a velocity to be specified as a boundary condition, since from equation 4-2 the velocity v_s in any direction s is given by:

$$v_s = - K \left(\frac{\partial \Psi}{\partial s} + \frac{\partial z}{\partial s} \right) \tag{4-8}$$

and $\frac{\partial z}{\partial s}$ is a constant.

The concept of the catchment as a unit requires the assumption that no flow crosses its boundaries except as precipitation or evaporation at the land surface, or as flow at the catchment outlet. So before discussing boundary conditions for a solution to represent the catchment the boundaries themselves must be specified.

A. <u>Catchment Surface</u>

This is the logical upper boundary for the system. The appropriate boundary condition will be the velocity of infiltration (using equation 4-8), which will be the current rainfall minus evaporation rate if the surface is unsaturated. If this rate exceeds the maximum (saturated) value of the hydraulic conductivity the surface will become saturated within a short time, but until it does all water arriving at the surface must be infiltrated. Water cannot exist on the surface of an unsaturated soil without a discontinuity in the value of Ψ at the surface, and infinite gradients of Ψ are not acceptable.

Once the surface becomes saturated, the arriving water must be divided between infiltration and additions to surface water in order to achieve a value of Ψ at the surface consistent with the depth of surface water. This is how the decrease of infiltration capacity with time is allowed for in the solution.

B. <u>Catchment Divide</u>

The surface concept of the catchment boundary as a line across which no flow occurs must be extended to a surface bounding the lateral subsurface flow within the catchment. For convenience this surface may be constructed by dropping verticals from all points on the surface boundary, although it must be recognised that subsurface flow is not always in the same direction as surface flow.

The boundary condition on this surface will be a zero flow, using equation 4-8 in the form $\frac{\partial \psi}{\partial s} = 0$.

C. Catchment Base

In many catchments an impermeable rock layer forms a lower boundary to the subsurface flow system. Hence a condition of zero flow at this depth must be imposed. This situation may also be thought of as modelling the decrease of conductivity with depth traditionally assumed to be responsible for rapid subsurface response. The choice will be made by selecting appropriate values of depth and soil properties (see Section 6.4).

Alternatively in cases where the groundwater level is relatively stable, specification of a zero pressure at this depth may be made. In this case the solution to Richards' equation would give the gradient of ψ at the water table, and equation 4-8 could be used to calculate the resultant inflow to groundwater.

D. Catchment Outlet

If the subsurface zone is to contribute to riverflow the channel must form part of the boundary of the system. The boundary condition will be a hydrostatic pressure distribution consistent with a solution to the equations of motion of the river subject to the subsurface and surface inflows from the catchment.

4.4 Configuration

The use of equation 4-6 to model the subsurface zone of a variety of catchments as required in a general model will only be possible if a standard configuration is chosen. This must then be fitted to a particular catchment by suitable choice of dimensions and soil properties. The boundaries of the complete, three-dimensional system were described in the last section but computational restraints may require that the system be modelled by a solution to some representative part of it. Possible configurations are listed below.

A. <u>Complete, Three-dimensional Solution</u>

Solution of Richards' equation for the complete, three-dimensional block of soil in which subsurface flow takes place would model the real system most closely (see Figure 4-4), but would be complex to solve even numerically, and require large amounts of data about the soil which would be impractical to obtain. The numerical solution would require the storage of information for each nodal point of at least a 20 x 20 x 20 grid system; only Freeze[29] has attempted such a solution, using a computer with unusually large core storage (1500K bytes). The details of coupling the solution with the equations of motion for surface flow have not yet been described. Use of this configuration will therefore not be justified until simpler solutions have been tried and found unsatisfactory.

B. <u>Two-dimensional Catchment Slice</u>

Use of a two-dimensional catchment "slice" to represent the subsurface catchment will reduce the complexity of the solution while retaining a dimension which can be used to depict areal variations (see Figure 4-5).

By choice of representative dimensions and average soil

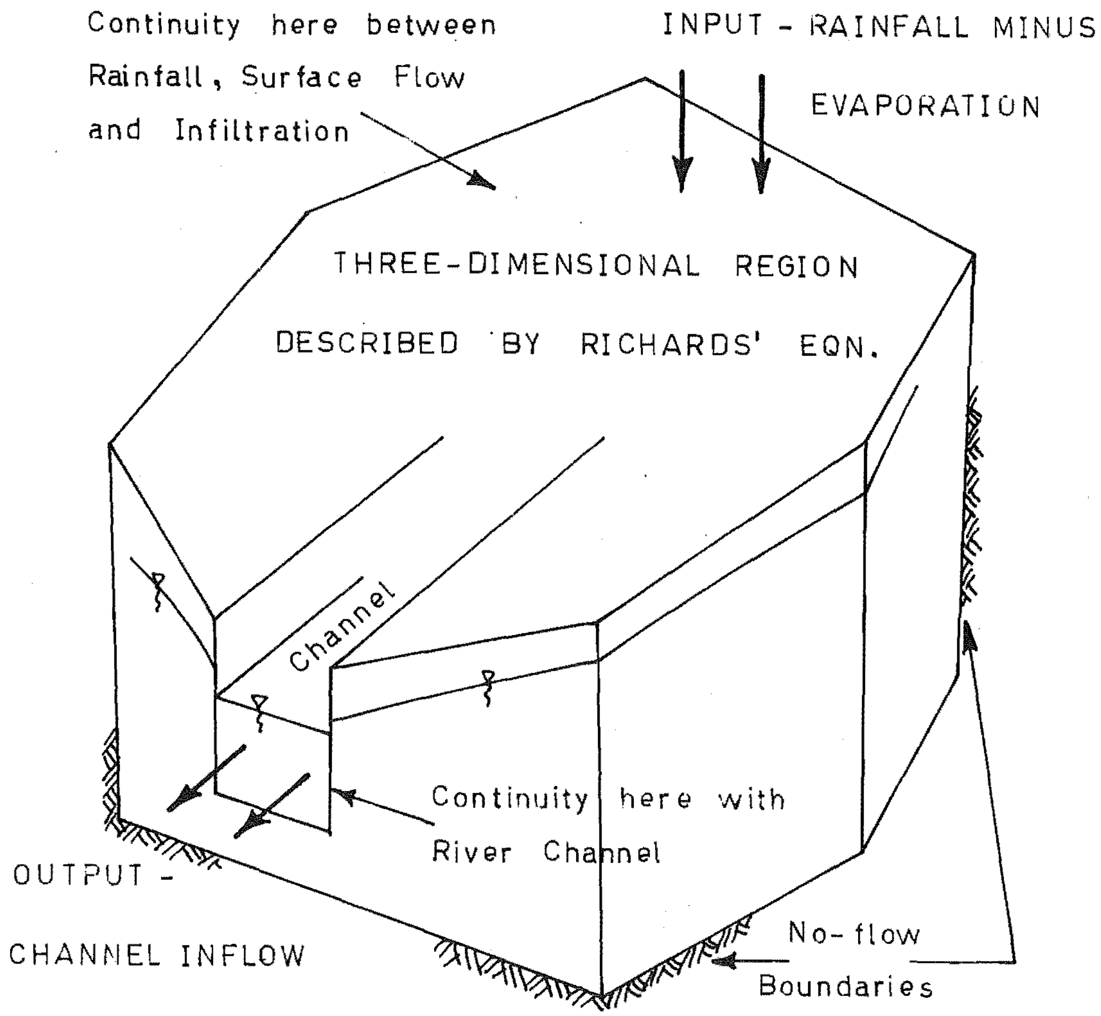


FIGURE 4-4: CONCEPTUAL VIEW OF THE SUBSURFACE FLOW SYSTEM

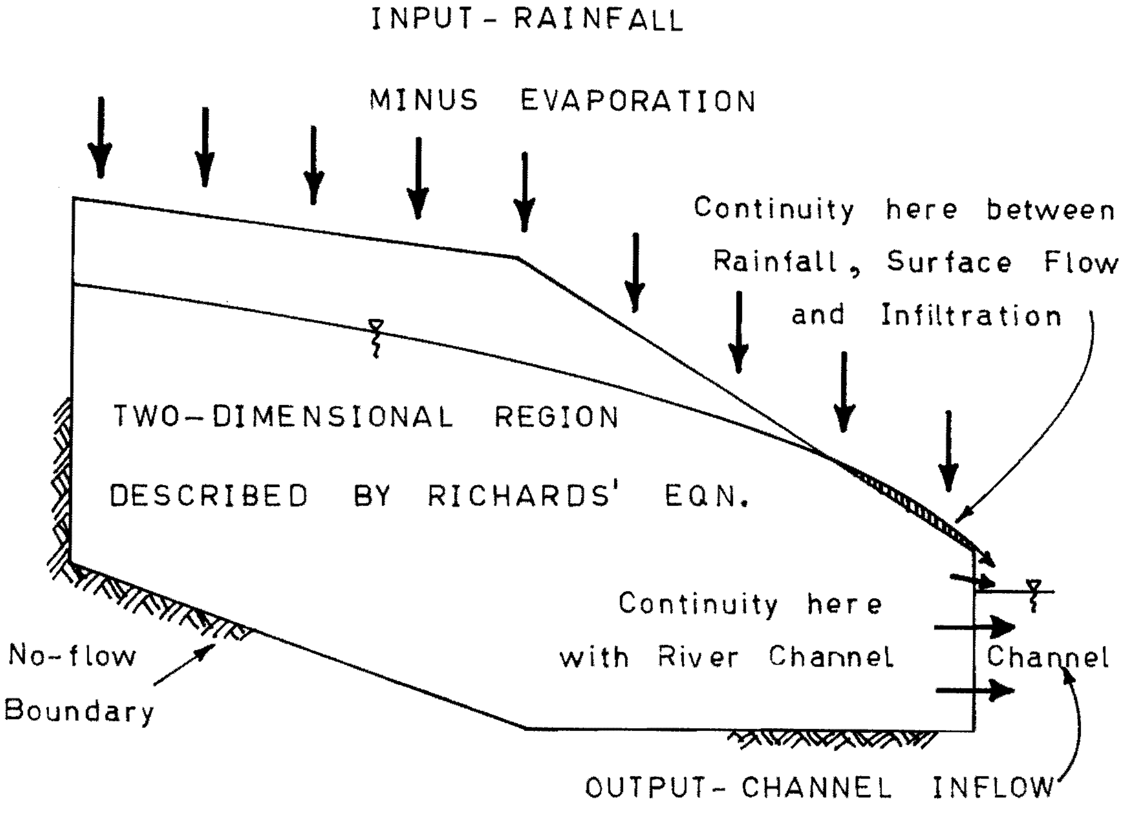


FIGURE 4-5: THE CATCHMENT SLICE

properties the input-output behaviour of a Richards' equation solution of a catchment slice could simulate many aspects of the behaviour of a real catchment. In particular it should be capable of simulating complete, partial-area or zero surface runoff by different choices of properties. Best results will be expected on Vee-shaped catchments whose properties on such cross-sections are similar throughout the catchment.

The numerical details of solving equation 4-6 in two dimensions have been described by several writers, and the coupling of such a solution to a solution of the equations of motion for riverflow has been achieved by Freeze[36]. But coupling to the equations of motion for surface flow has so far been limited to the one-dimensional form of Richards' equation. If a 20 x 20 nodal grid is the minimum for a two-dimensional solution, this would require 20 times the computer storage of a one-dimensional solution. As for the three-dimensional solution the amount of soil data required is extensive, and any attempt to estimate the soil properties by trial-and-error would be difficult because of the large number of parameters. The problems yet to be solved indicate that this approach would best be tackled via the stepping-stone of the simplest, one-dimensional solution, at least for a model to be used on real catchments.

C. One-dimensional Catchment Column

Many catchment models are lumped; that is, a distributed property of the catchment is depicted by a single, representative value of a model parameter, chosen so that the behaviour of model and catchment are as close as possible. The representation of the subsurface zone of a catchment by a one-dimensional, vertical "column" of soil relies on this concept. The error introduced by this approach will depend on the degree of non-uniformity of the catchment.

In this approach the subsurface behaviour is modelled by solving the one-dimensional form of equation 4-6 for a vertical column of soil situated at a "typical" point in the catchment (see Figure 4-6). The solution could operate in the following manner:

(a) The solution at the previous time step in terms of the distribution of moisture (and hence pressure or suction) along the vertical will form the initial condition for a calculation. This is the way in which antecedent moisture enters into the solution.

(b) The boundary condition at the upper end of the column will be the current infiltration at the surface (see Section 4.3A). This is the way in which climate, and especially rainfall, affects the solution.

(c) The boundary condition at the lower end of the column must represent a velocity if the column is to contribute to riverflow. If this velocity is made to depend on the moisture conditions within the column, which in turn depend on the rainfall history, then the column can be made to exhibit the desired catchment-like input-output behaviour.

(d) Solution of equation 4-6 subject to these initial and boundary conditions will yield an updated distribution of moisture ready for the next time step. This updated moisture distribution will define a new lower boundary velocity which may be used to represent a subsurface contribution to riverflow.

Freeze[28] describes a one-dimensional solution to

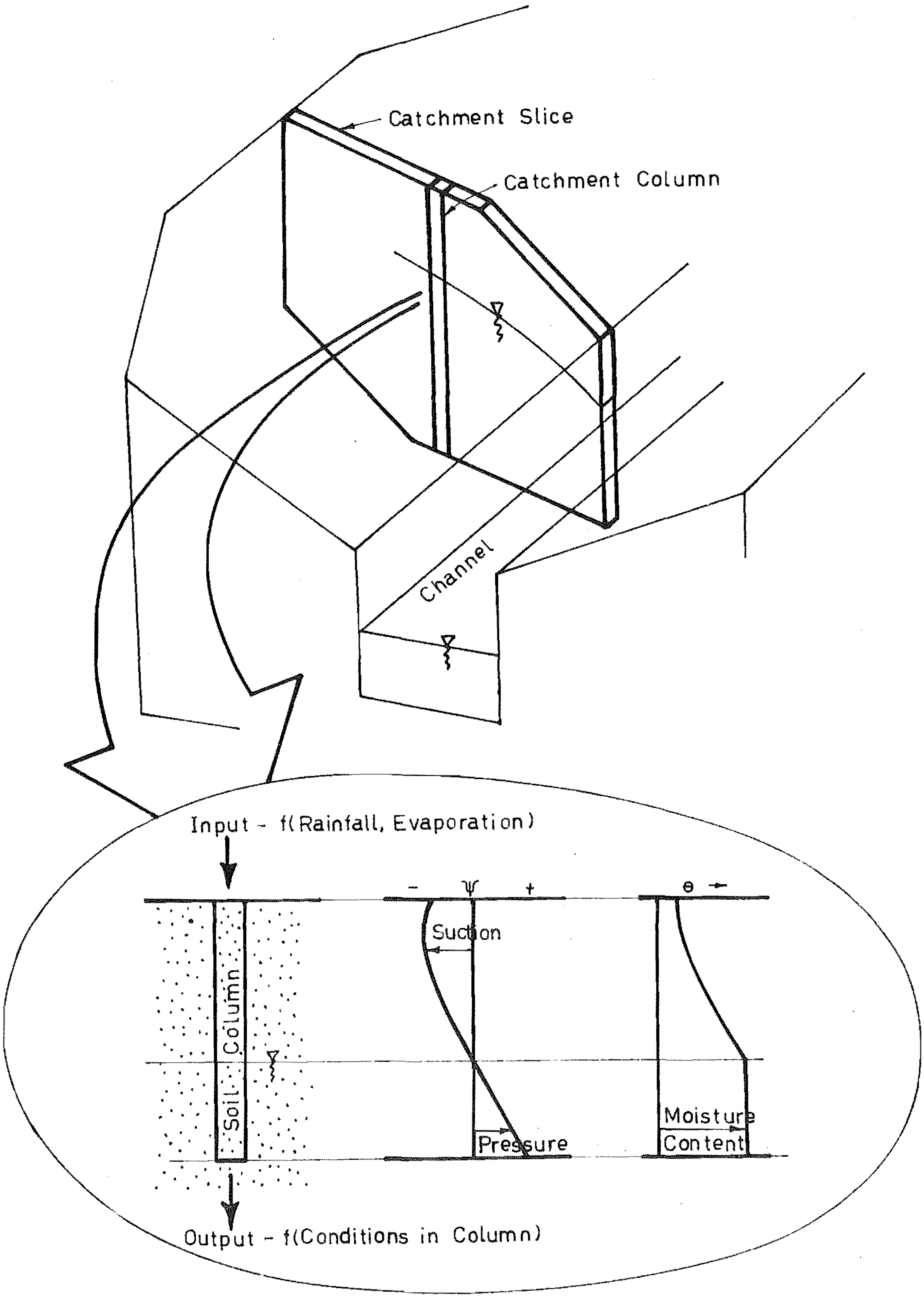


FIGURE 4-6: THE CATCHMENT COLUMN IN ITS CONTEXT

equation 4-6, in which the distribution of ψ is obtained in response to boundary conditions imposed as velocities at the ends of a soil column. The distribution of moisture is obtained from this via the moisture content-soil suction relation for the soil. An important feature of Freeze's solution is the simultaneous treatment of the saturated and unsaturated portions of the column, which enables the boundary between them (the water table) to be predicted as it rises and falls in accordance with the boundary conditions (see Figure 4-7). The numerical details of a soil column solution suitable for representing the subsurface zone of a catchment, based on that of Freeze, appear in Section 5.4.

The column solution has the advantage of comparative simplicity at the expense of some degree of physical reality, compared with the catchment slice and the complete three-dimensional solutions. The expected advantages which a better subsurface description was expected to confer on the SWM (Section 3.5) are now re-examined with the column solution in mind.

(a) Physical Reality

Although inferior to a scheme which takes areal variation into consideration the column solution is thought to be more realistic than the empirical relations of the SWM.

(b) Partial Area Runoff

The column solution cannot simulate partial area runoff; the upper boundary of the column will either be saturated, in which case there will be water available for surface flow, or it will be unsaturated and no surface flow can occur.

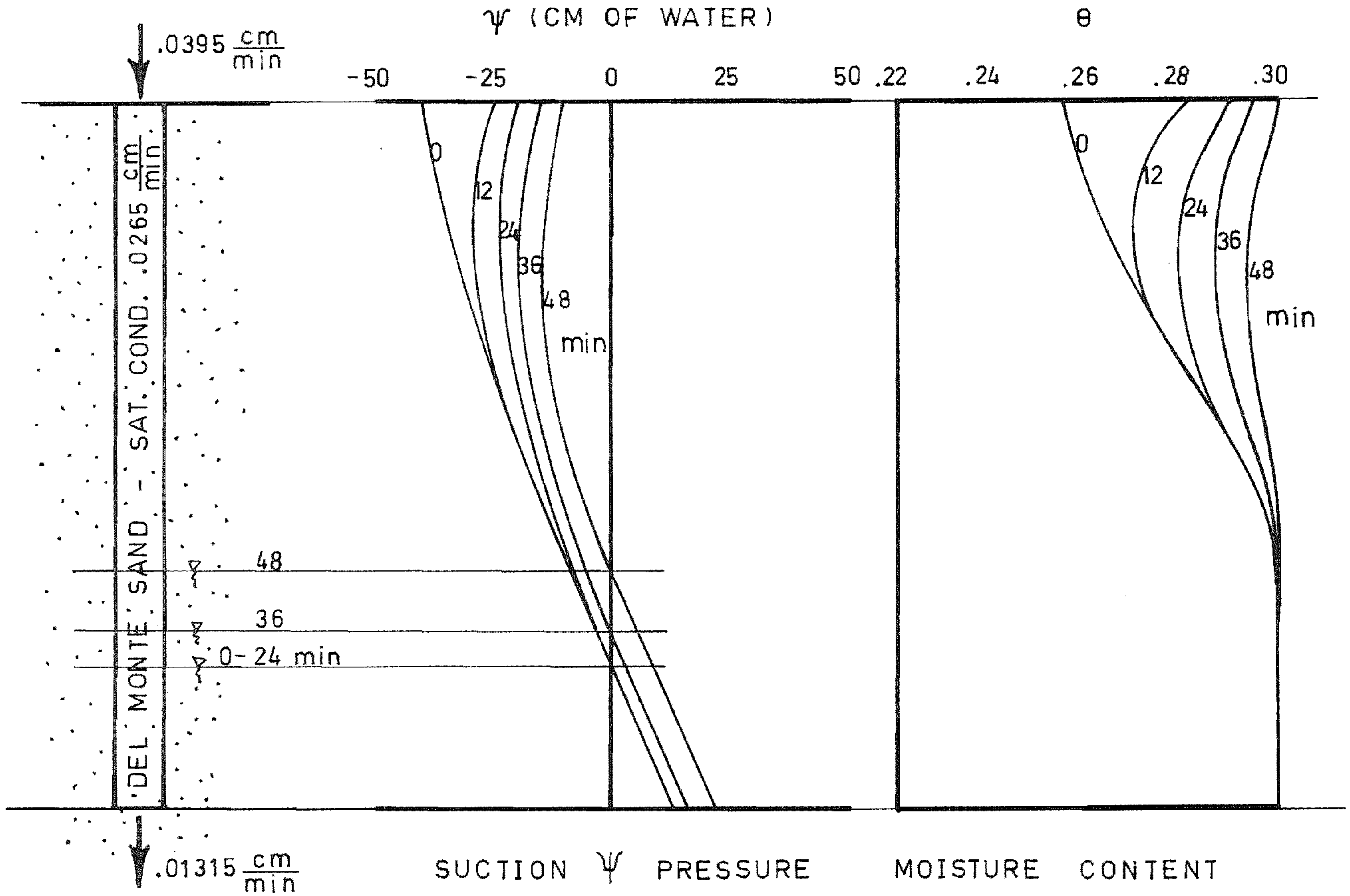


FIGURE 4-7: FREEZE'S SOIL COLUMN SOLUTION

(c) Component Processes

The column solution could still replace all the subsurface flow components, including infiltration, of the SWM.

(d) Areal Variation

The column solution cannot improve on the areal lumping of the SWM.

(e) Parameter Estimation

Subject to the difficulty of sampling a heterogeneous catchment, the soil column properties should be more readily assessed than the parameters of the SWM.

In spite of the severe restrictions involved in the one-dimensional assumption, three areas of potential improvement exist. Therefore as well as forming a base on which a potentially superior model could be built, the aim of this study (Section 3.5) may be partly fulfilled by the replacement of the subsurface components of the SWM by a one-dimensional solution to Richards' equation. The performance of this Amended Model may then be used in conjunction with a review of more complex solutions to Richards' equation to predict the utility of pursuing this approach.

CHAPTER FIVE

THE AMENDED MODEL

5.1 The Amendments

The aim of this study is to determine whether the Richards' equation description of flow in the subsurface zone of a catchment will improve the performance of the SWM. Firstly the subsurface components of the SWM are replaced by a solution of Richards' equation for a one-dimensional, vertical column of soil. Secondly the performance of this Amended Model (AM), and the restrictions which the one-dimensional assumption involves, are examined in conjunction with the features of more general two- and three-dimensional solutions to Richards' equation. This chapter describes the first of these parts, the replacement by the soil column solution of the subsurface components of the SWM.

Figure 5-1 is a block diagram of the structure of the SWM with the subsurface components enclosed within a dotted line. These components represent infiltration, prompt and delayed subsurface flow. Section 4.4C discussed how both subsurface flow and infiltration might be represented by the soil column solution. This solution was inserted in place of the components inside the dotted line of Figure 5-1 to create the Amended Model, the structure of which is shown in Figure 5-2. Thus the components of the AM, which are described in detail in the remainder of this chapter, are:

(a) Interception, which is modelled by a fixed-capacity

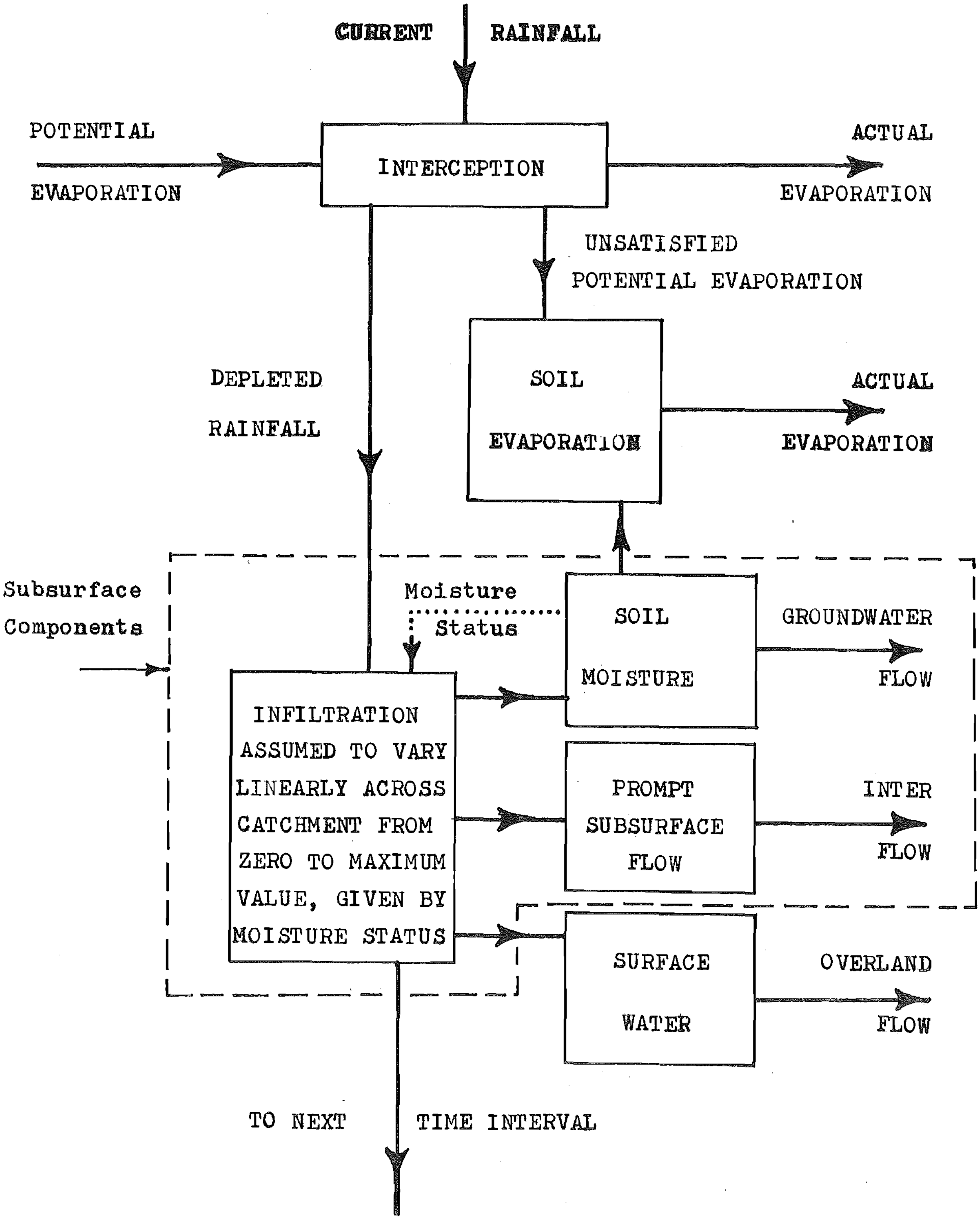


FIGURE 5-1: THE STRUCTURE OF THE STANFORD WATERSHED MODEL

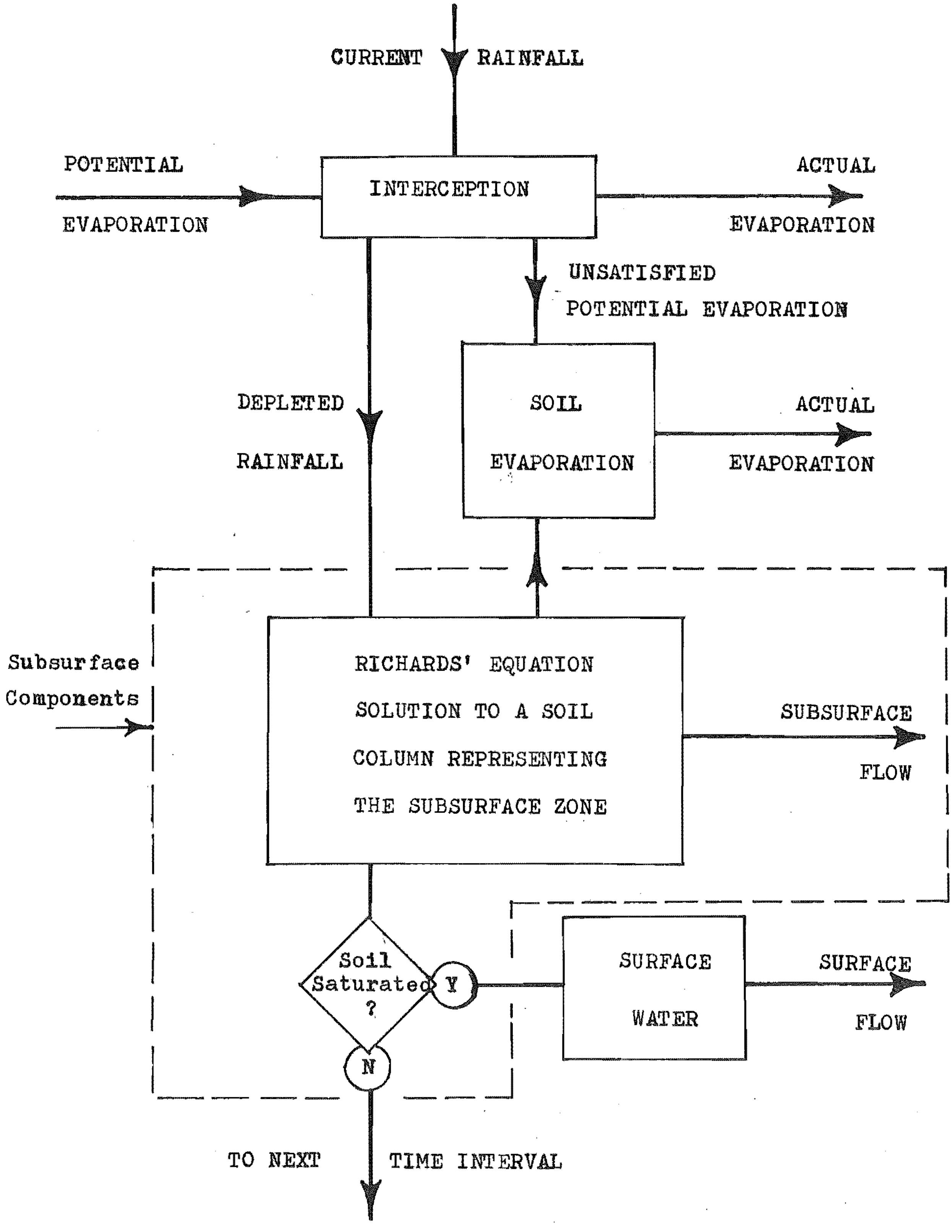


FIGURE 5-2: THE STRUCTURE OF THE AMENDED MODEL

storage. All rainfall up to the water deficit at any time, representing the interception capacity of the vegetation, is retained by the storage for later evaporation; the excess is available for infiltration.

(b) Evaporation, which is based on daily or monthly estimates of potential evapotranspiration. Water in the interception storage or on the land surface is allowed to evaporate up to the current potential rate. Any unsatisfied potential evapotranspiration is then allowed to extract water from the soil at a reduced rate.

(c) Infiltration and Subsurface Flow, which are modelled by the numerical solution to Richards' equation applied to a one-dimensional, vertical column of soil representing the subsurface zone of the catchment. The boundary condition at the top of the column corresponding to the land surface is the rainfall minus interception and soil evaporation; at the bottom of the column representing subsurface channel input the boundary condition is a velocity proportional to the height of the water table.

(d) Surface Flow, which is depicted by a solution of the equations of motion for flow over a sloping plane adapted to suit a lumped model. This component operates whenever the soil column solution predicts that there is water on the land surface.

Because the study is restricted to hydrologically small catchments the riverflow component of the SWM was not used nor was it incorporated into the AM. Volumes of flow predicted each time step from the subsurface and surface flow components are added and assumed to become riverflow at the outlet during that time step. Similarly, because snowmelt is excluded from this study, the SWM snowmelt component was not included in the AM. However the interception, evaporation and surface flow components in the AM are substantially the same as those of the SWM.

The numerical operations for each of these components, which are described in the remainder of this chapter, are carried out by computer. Input to the computer program consists of soil and other catchment properties, the initial moisture condition of the catchment and records of rainfall, evapotranspiration and riverflow for a given period of time. The program output comprises a graph of the riverflow simulated by the model overprinted on a graph of recorded flow, various measures of the agreement between these graphs, and the final moisture condition of the catchment. The program listing, input requirements and a sample output are given in Appendix A.

The equations describing these processes are generally solved for each time step for which the input and output data are defined. However, the program has the ability to combine several time steps into one during times of relative inactivity in order to reduce computing time. "Times of inactivity" are those when there is no rainfall and the soil water table is lower than a specified level.

5.2 Interception

Interception of rainfall by vegetation is represented by a storage whose size depicts the average depth of water which the vegetation on the catchment is capable of holding (see Figure 5-3). At a given time the depth of water in the storage represents intercepted water while the remaining

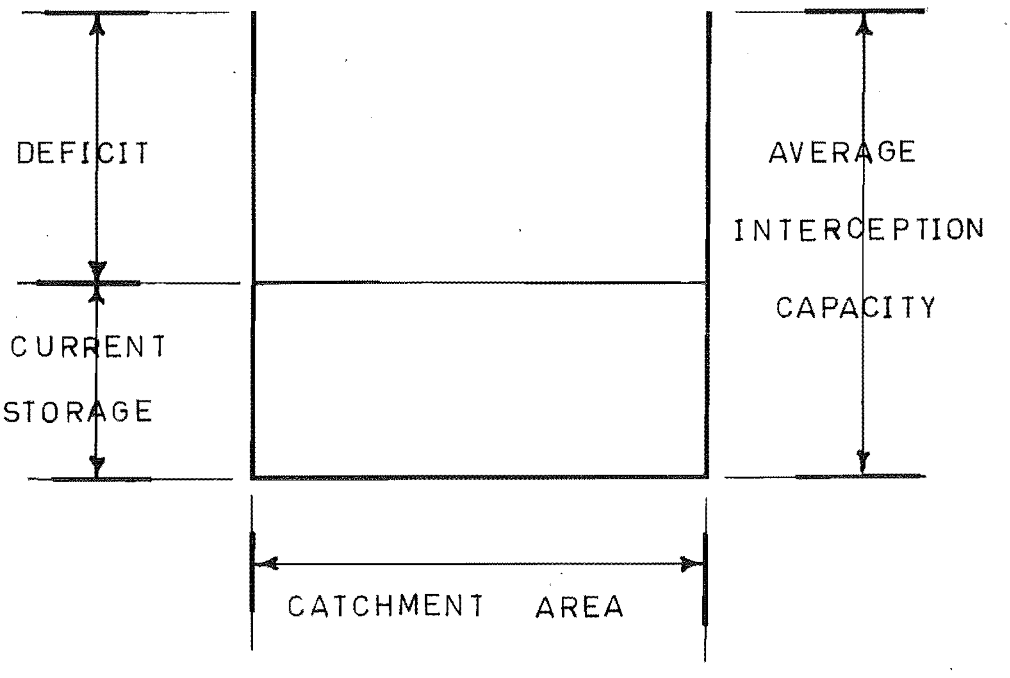


FIGURE 5-3: THE INTERCEPTION COMPONENT

depth, referred to below as the deficit, represents capacity to intercept water from further rain.

In each time interval of the simulation the rainfall depth is added to the water currently in the interception storage. If the rainfall exceeds the deficit the surplus is available for input to the next component, infiltration. Thus the amount of rainfall subtracted by the interception component depends both on the storage size and on the antecedent condition as defined by the deficit. The relation between input (rainfall) and output (surplus available for infiltration) is shown in Figure 5-4.

5.3 Evaporation

Pan evaporation data or Penman evaporation estimates are used to calculate daily potential evapotranspiration values. These values are distributed throughout the day according to a cosine-curve which approximates the actual evapotranspiration observations of van Bavel[38]; this relation is given by:

$$\begin{aligned} e &= 0 && t \leqslant 3 \\ e &= \frac{E}{21}\left\{1 - \cos\frac{2\pi(t-3)}{21}\right\} && t > 3 \end{aligned} \qquad (5\text{-}1)$$

where e is the instantaneous evapotranspiration rate,

E is the total evapotranspiration for the day, and

t is the time in hours from midnight.

Van Bavel's observations and equation 5-1, reduced to unit daily volume, are shown in Figure 5-5.

During each time step of the simulation, the potential evapotranspiration is allowed to remove water first from the

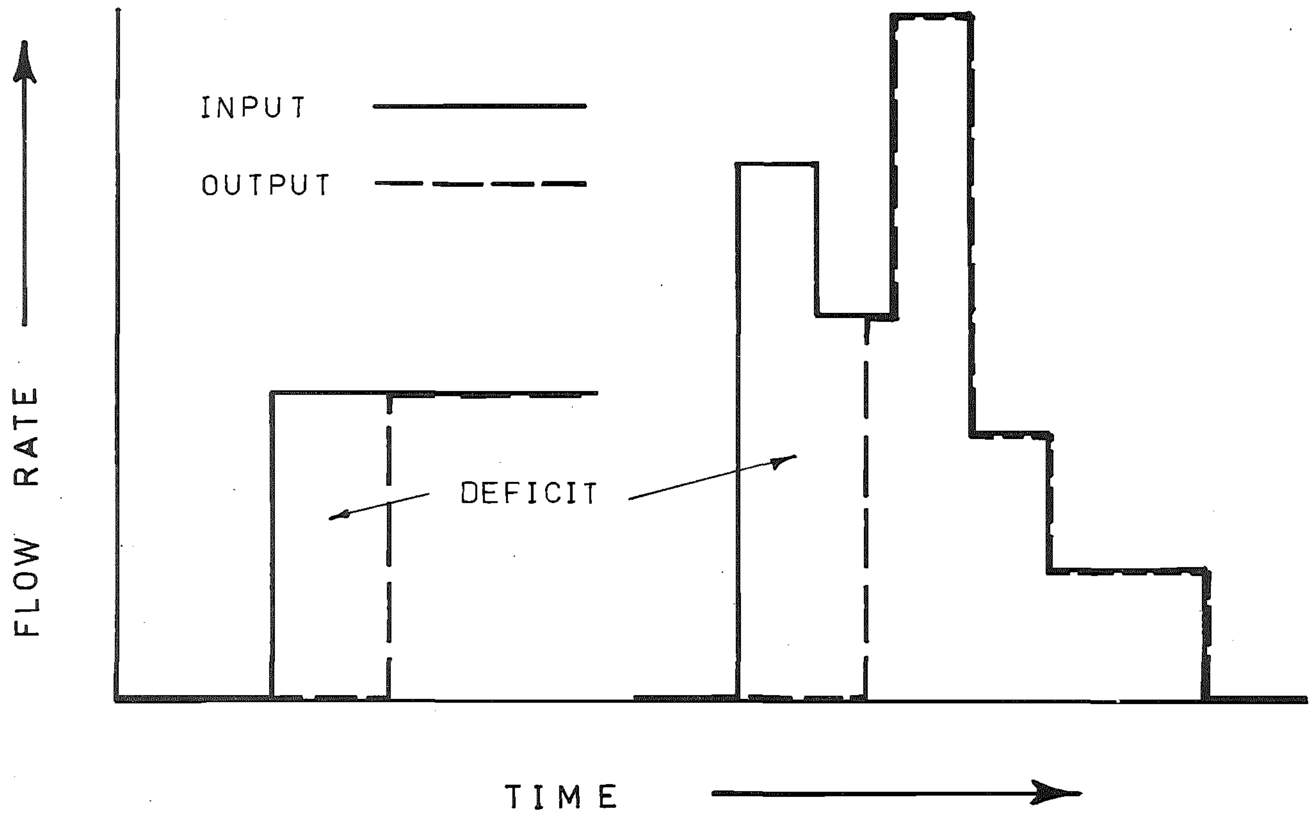


FIGURE 5-4: INPUT-OUTPUT BEHAVIOUR OF THE INTERCEPTION COMPONENT

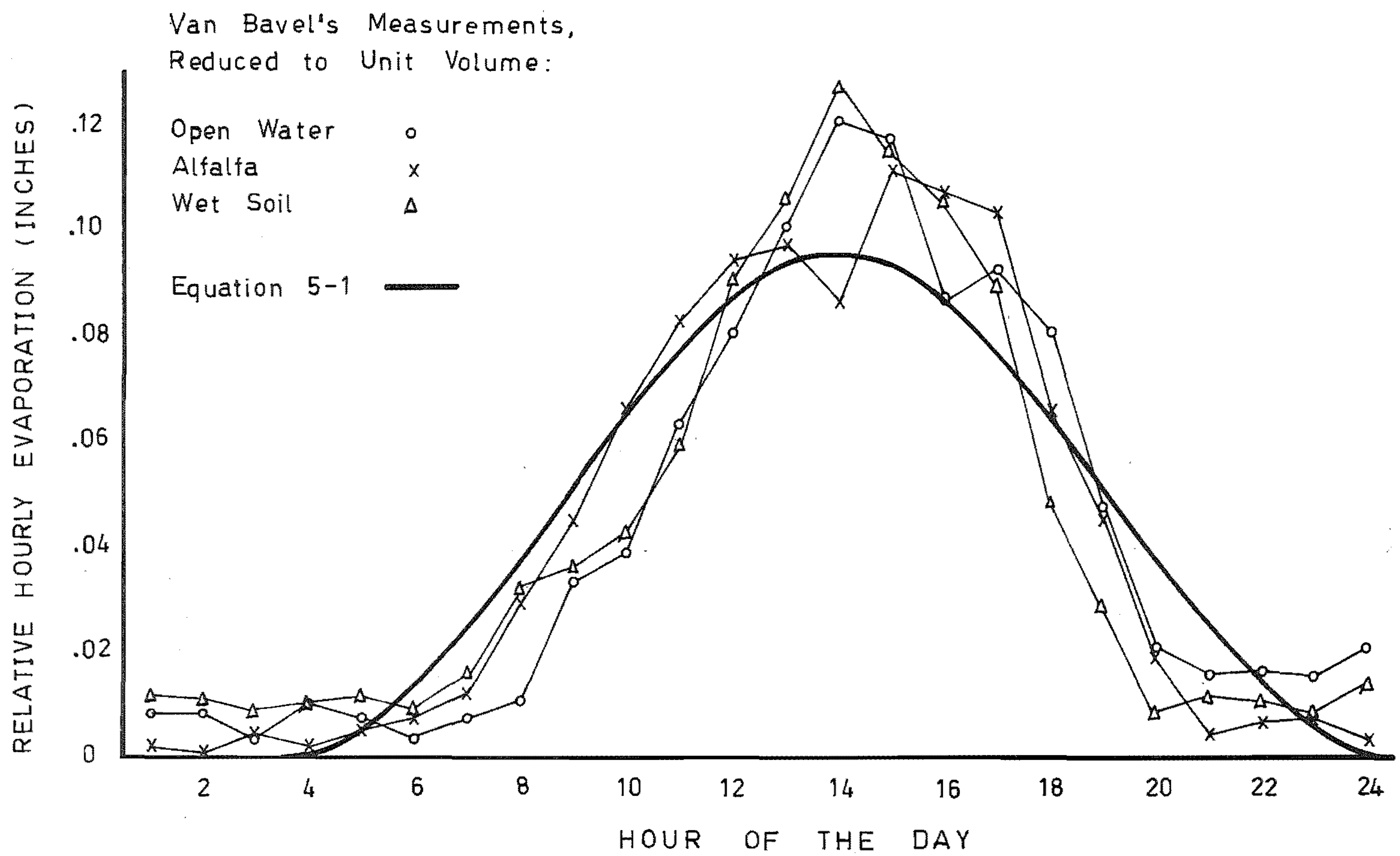


FIGURE 5-5: VARIATION OF EVAPOTRANSPIRATION THROUGHOUT THE DAY

interception storage, and then from the land surface. Any unsatisfied potential is then modified according to the method of Boughton[18], in which the actual evapotranspiration is reduced below the potential rate when the soil becomes dry.

Boughton's method, illustrated in Figure 5-6, is based on transpiration measurements of Denmead and Shaw[39], who found that transpiration can occur at the potential rate down to very low soil moisture contents provided the potential rate is low. But when the potential rate is high the transpiration rate reduces as the soil dries out. The reduced evapotranspiration thus found contributes to the upper boundary condition for the soil column solution.

The action of evapotranspiration influences the behaviour of the interception component, whose input-output behaviour with a constant evaporation rate is shown in Figure 5-7. This action is analogous to the concept of an initial plus continuing "loss".

5.4 Infiltration and Subsurface Flow

A. General

Both infiltration and subsurface flow are represented by a numerical solution to Richards' equation applied to a one-dimensional, vertical column of soil. The way in which this solution models the subsurface zone of a catchment is given below; the remainder of this section describes the numerical details of the solution, the boundary and initial conditions under which the equation is solved, the input-output behaviour and the assumptions made in applying the column solution to a catchment model.

The volume of soil which is thought to influence flow

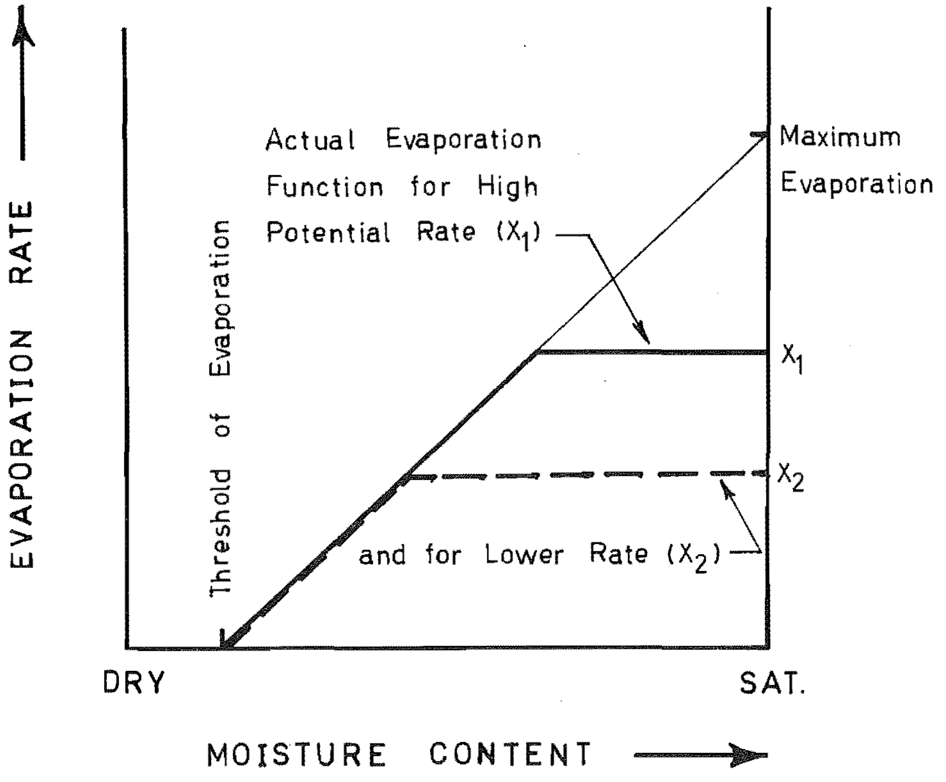


FIGURE 5-6: BOUGHTON'S METHOD FOR CALCULATING ACTUAL FROM POTENTIAL EVAPOTRANSPIRATION

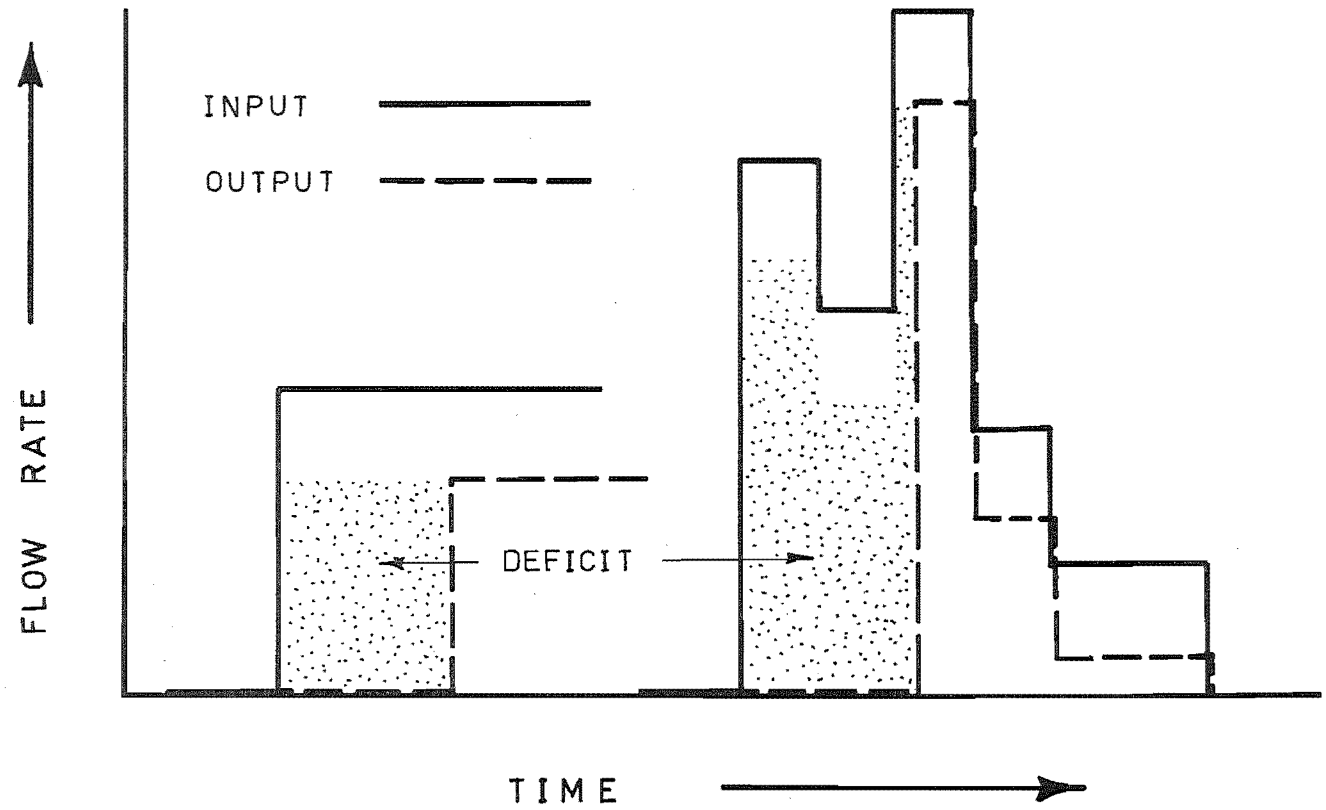


FIGURE 5-7: INPUT-OUTPUT BEHAVIOUR OF THE INTERCEPTION COMPONENT SUBJECT TO A CONSTANT EVAPORATION

at a chosen point in a river was shown in Figure 4-4; ideally Richards' equation should be solved for this entire region. But the difficulty of measuring soil properties, of solving the equation and of ensuring the solution is compatible with the solutions to surface and channel flow dictate that the block of soil must be represented by a soil slice (see Figure 4-5), and again that the slice must be represented by a column of soil (see Figure 4-6). A detailed discussion of this idealisation appears in Section 4.4C.

The solution of Richards' equation for a single column of soil cannot describe the variation in properties or behaviour which occurs over a catchment. But by suitably choosing the properties and dimensions of the column its solution can be made to approximate the behaviour of the subsurface zone as a whole, in accepting or rejecting rainfall, and in storing, evaporating or transmitting infiltrated water to the river.

The formulation used for the solution to Richards' equation was based on the work of Freeze[28], whose solution was obtained as the distribution of pressure or suction head ψ at successive time intervals, in response to boundary conditions imposed as velocities at the ends of the column. An important feature of Freeze's solution was the continuous treatment of the saturated and unsaturated portions of the column, which enabled the boundary between them (the water table) to be predicted as it rose and fell in accordance with the boundary conditions (see Figure 4-7). In particular saturation of the upper end of the column representing the land surface could be predicted, thus enabling the simulation of the occurrence of surface flow. After modifications to

Freeze's solution to improve the numerical stability, to remove an inconsistency in the logic controlling saturation and to preserve continuity between the column and other model components, the following soil column solution was adopted for use in the AM.

B. One-dimensional Numerical Solution to Richards' Equation

(a) The One-dimensional Equation

The one-dimensional, vertical form of equation 4-6 is:

$$C \frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z} \left\{ K \left(\frac{\partial \psi}{\partial z} + 1 \right) \right\} \tag{5-2}$$

The boundary condition for the specification of a velocity v_z in the vertical direction is, from equation 4-8:

$$v_z = -K \left(\frac{\partial \psi}{\partial z} + 1 \right) \tag{5-3}$$

Equation 5-2 is a nonlinear, parabolic, partial differential equation; to ensure stability an implicit, backward-time finite-difference scheme is used. The solution is carried out in terms of the soil suction ψ, which may be positive or negative, and each time step yields an updated distribution of ψ in response to the boundary conditions imposed by equation 5-3. This ψ-distribution may be converted to a moisture content distribution via the moisture content-soil suction relation for the porous medium.

To start the calculation an initial array of suctions must be specified. In addition the relations between soil suction, hydraulic conductivity and moisture content are required in the course of the solution. Although in principle hysteresis between the wetting and drying curves can be accommodated in the solution, this would require the storage of the history of each point in the soil to determine whether it is wetting or drying at each instant, and is not done here. This means the variables K and θ are functions of ψ alone. The specific moisture capacity C follows as the slope of the moisture content-soil suction curve.

(b) Finite-difference Scheme

Equations 5-2 and 5-3 are approximated by a backward-time finite-difference scheme. Referring to Figure 5-8, the equations become:

For a general interior point

$$C\left(\psi_j^{t-\frac{1}{2}}\right)\frac{\psi_j^t-\psi_j^{t-1}}{\Delta t}=\frac{1}{\Delta z}\left\{\begin{array}{l}K\left(\psi_{j+\frac{1}{2}}^t\right)\left(\dfrac{\psi_{j+1}^t-\psi_j^t}{\Delta z}+1\right)\\[2ex] \quad -K\left(\psi_{j-\frac{1}{2}}^t\right)\left(\dfrac{\psi_j^t-\psi_{j-1}^t}{\Delta z}+1\right)\end{array}\right\} \tag{5-4}$$

And for the boundary points

$$-v_{LB}=K\left(\psi_{j+\frac{1}{2}}^t\right)\left(\frac{\psi_{j+1}^t-\psi_j^t}{\Delta z}+1\right) \tag{5-5}$$

$$-v_{UB}=K\left(\psi_{j-\frac{1}{2}}^t\right)\left(\frac{\psi_j^t-\psi_{j-1}^t}{\Delta z}+1\right) \tag{5-6}$$

where ψ_j^t is the value of ψ at distance step j and time step t,

v_{LB} is the velocity at the lower boundary,

v_{UB} is the velocity at the upper boundary, and

Δz and Δt are the distance and time steps.

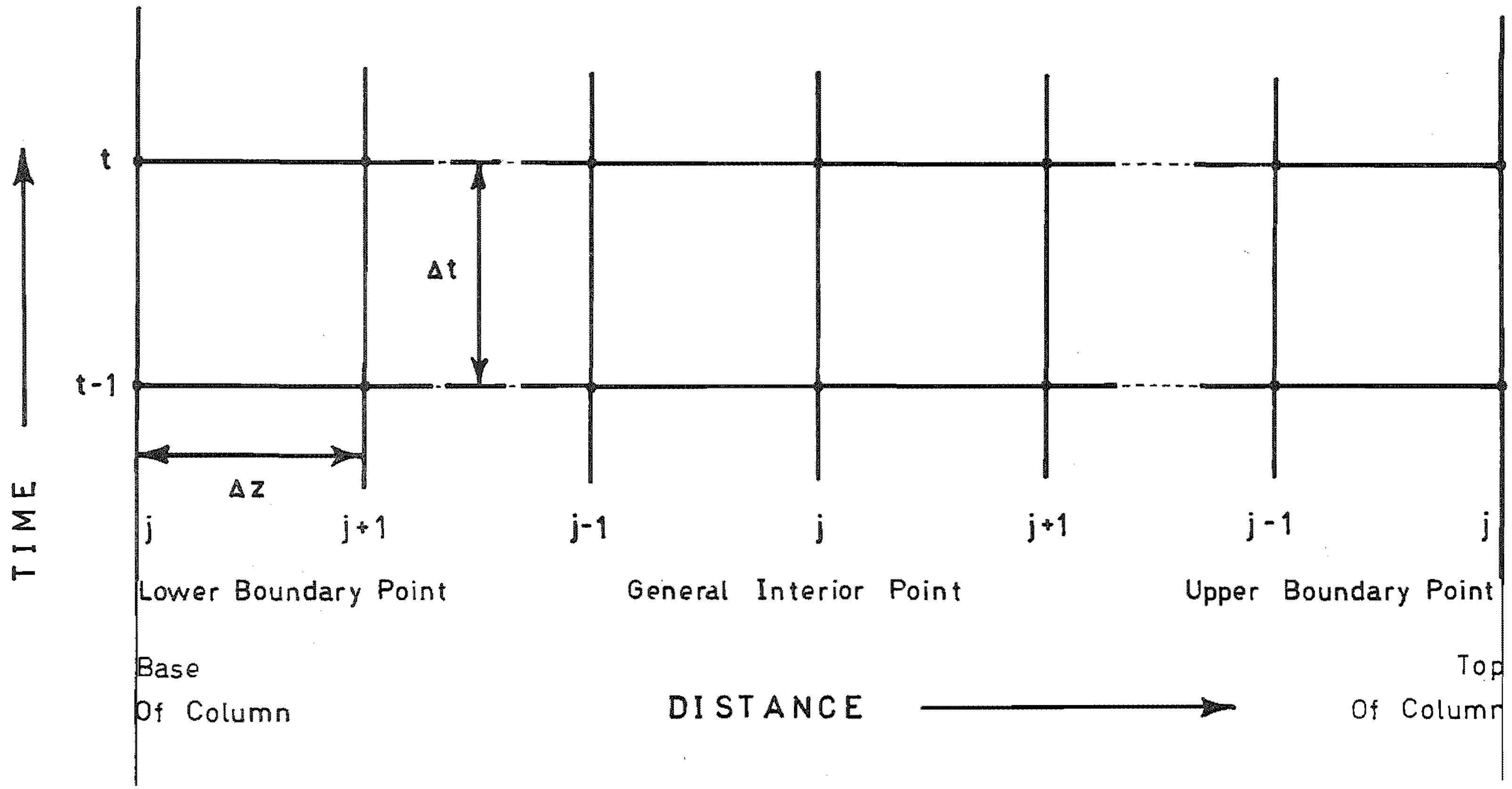


FIGURE 5-8: DEFINITION SKETCH FOR FINITE-DIFFERENCE APPROXIMATIONS TO EQUATIONS 5-2 AND 5-3

Adoption of the backward-time difference scheme solved problems earlier experienced with a central-difference (Crank-Nicholson) scheme which seem to be similar to those experienced by Freeze[29]. Under steady-state boundary conditions oscillations occurred in the solution which could not be removed by increasing the precision with which the calculations were carried out or by reducing the time step. Both schemes are said to be unconditionally stable[37], but it is thought that the problem originated with the non-constancy of the coefficients C and K.

The coefficients $K(\psi_{j+\frac{1}{2}}^{t})$, $K(\psi_{j-\frac{1}{2}}^{t})$ and $C(\psi_{j}^{t-\frac{1}{2}})$ depend on the values of ψ yet to be found for the new time t; so an iterative process should strictly be used, starting with the known values $K(\psi_{j+\frac{1}{2}}^{t-1})$, $K(\psi_{j-\frac{1}{2}}^{t-1})$ and $C(\psi_{j}^{t-1})$ as first approximations. However, provided that ψ is not varying too rapidly with time, the first approximation may be close enough and the iteration dispensed with. Under these circumstances, equations 5-4, 5-5 and 5-6 become:

For a general interior point

$$C(\psi_j^{t-1})\,\frac{\psi_j^t-\psi_j^{t-1}}{\Delta t}=\frac{1}{\Delta z}\left\{\begin{array}{l} K(\psi_{j+\frac{1}{2}}^{t-1})\left(\dfrac{\psi_{j+1}^t-\psi_j^t}{\Delta z}+1\right) \\ \qquad -K(\psi_{j-\frac{1}{2}}^{t-1})\left(\dfrac{\psi_j^t-\psi_{j-1}^t}{\Delta z}+1\right)\end{array}\right\} \qquad (5\text{-}7)$$

And for the boundary points

$$-V_{LB}=K(\psi_{j+\frac{1}{2}}^{t-1})\left(\frac{\psi_{j+1}^t-\psi_j^t}{\Delta z}+1\right) \qquad (5\text{-}8)$$

$$-V_{UB}=K(\psi_{j-\frac{1}{2}}^{t-1})\left(\frac{\psi_j^t-\psi_{j-1}^t}{\Delta z}+1\right) \qquad (5\text{-}9)$$

Now that all the coefficients C and K have been expressed in terms of the known values of ψ at time t-1, equations 5-7, 5-8 and 5-9 may be rearranged to form a set of simultaneous linear algebraic equations in the unknown values of ψ at time t:

For a general interior point

$$\psi_{j+1}^{t}\left\{\frac{K(\psi_{j+\frac{1}{2}}^{t-1})}{\Delta z^2}\right\} - \psi_{j}^{t}\left\{\frac{C(\psi_{j}^{t-1})}{\Delta t} + \frac{K(\psi_{j+\frac{1}{2}}^{t-1}) + K(\psi_{j-\frac{1}{2}}^{t-1})}{\Delta z^2}\right\} + \psi_{j-1}^{t}\left\{\frac{K(\psi_{j-\frac{1}{2}}^{t-1})}{\Delta z^2}\right\}$$

$$= \left\{\frac{K(\psi_{j-\frac{1}{2}}^{t-1}) - K(\psi_{j+\frac{1}{2}}^{t-1})}{\Delta z} - \frac{C(\psi_{j}^{t-1})\,\psi_{j}^{t-1}}{\Delta t}\right\} \tag{5-10}$$

And for the boundary points

$$\psi_{j+1}^{t}\left\{\frac{K(\psi_{j+\frac{1}{2}}^{t-1})}{\Delta z}\right\} - \psi_{j}^{t}\left\{\frac{K(\psi_{j+\frac{1}{2}}^{t-1})}{\Delta z}\right\} + \psi_{j-1}^{t}\left\{\ 0\ \right\}$$

$$= \left\{-V_{LB} - K(\psi_{j+\frac{1}{2}}^{t-1})\right\} \tag{5-11}$$

$$\psi_{j+1}^{t}\left\{\ 0\ \right\} - \psi_{j}^{t}\left\{\frac{K(\psi_{j-\frac{1}{2}}^{t-1})}{\Delta z}\right\} + \psi_{j-1}^{t}\left\{\frac{K(\psi_{j-\frac{1}{2}}^{t-1})}{\Delta z}\right\}$$

$$= \left\{V_{UB} + K(\psi_{j-\frac{1}{2}}^{t-1})\right\} \tag{5-12}$$

Equations 5-10, 5-11 and 5-12 have the general form:

$$A\,\psi_{j+1}^{t} - B\,\psi_{j}^{t} + C\,\psi_{j-1}^{t} = D \tag{5-13}$$

in which the coefficients A, B, C and D are all functions of the boundary velocities imposed, or of the conditions at the previous time step. Solution of the simultaneous equations by Gauss Elimination and back-substitution yields the values of the ψ_{j}^{t}. The form of the solution is illustrated in Figure 5-9.

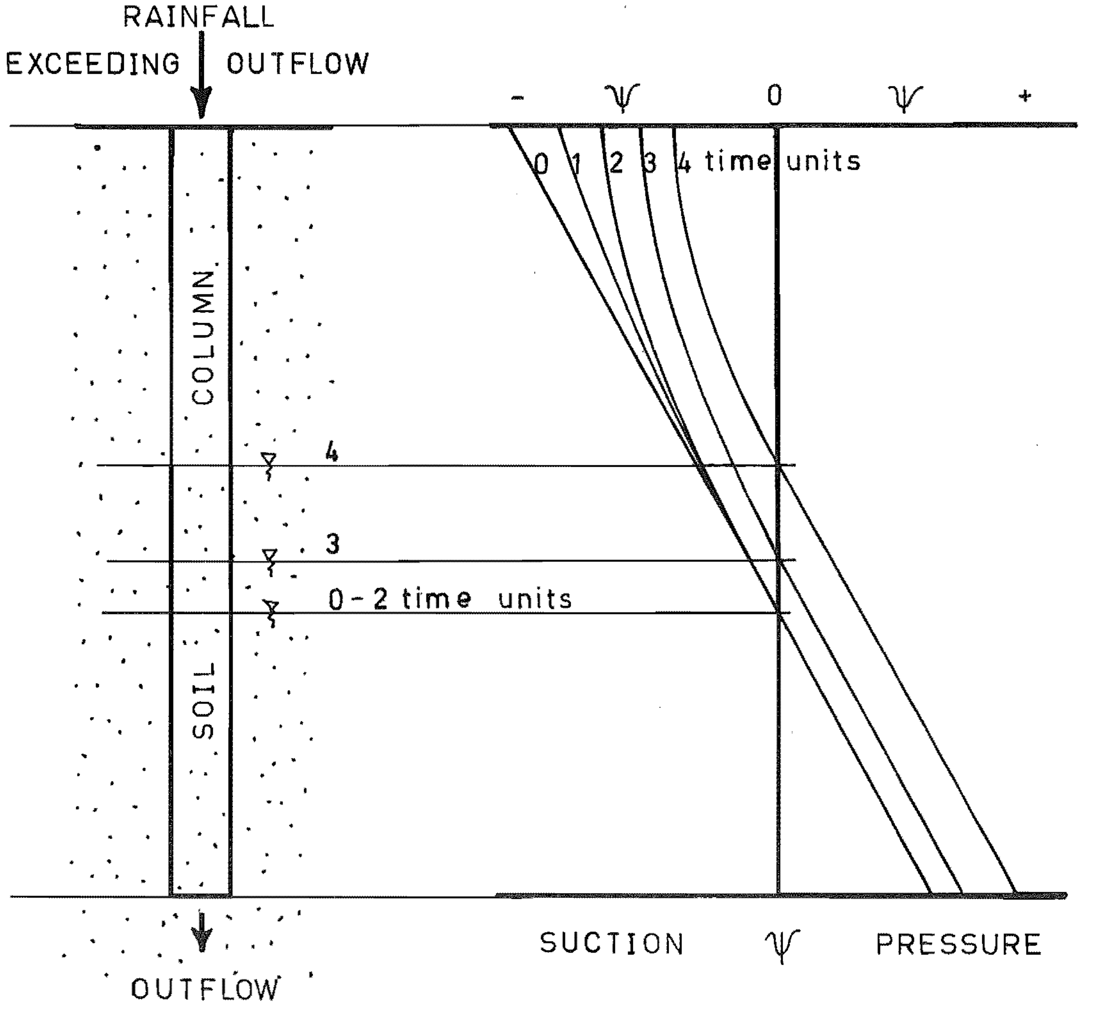


FIGURE 5-9: SATURATION FROM BELOW

The operations described above for one time step were programmed as a subroutine for digital computer solution (see Subroutine SOIL, Appendix A). Input to the subroutine consists of soil properties, initial conditions and the current boundary velocities, and the output is the updated distribution of suction and moisture content.

(c) Saturation

An important feature of the solution is the ability to predict the fluctuating position of the water table. This is inferred as the point or points where $\psi = 0$, and these rise and fall in response to the specified boundary conditions. Solution within the saturated zone is carried out just as for the unsaturated zone, the only difference being that the coefficients C and K assume their constant saturated values.

Saturation from below via a rising water table occurs when the lower boundary velocity is less than the upper boundary velocity. The column cannot transmit the water as rapidly as it arrives so the soil "fills up" from the bottom (see Figure 5-9).

Saturation from above occurs when the upper boundary velocity exceeds the saturated hydraulic conductivity. In this case the surface saturates after a short time and an inverted water table propagates down into the soil (see Figure 5-10). The prediction by the solution of a positive value of ψ (a pressure) at the surface implies that there exists an equivalent depth of water on the surface. But surface water can only occur if there is a difference between the water arriving at the surface (e.g. rainfall) and water infiltrating. Therefore when this happens the upper boundary velocity must be reduced until continuity is

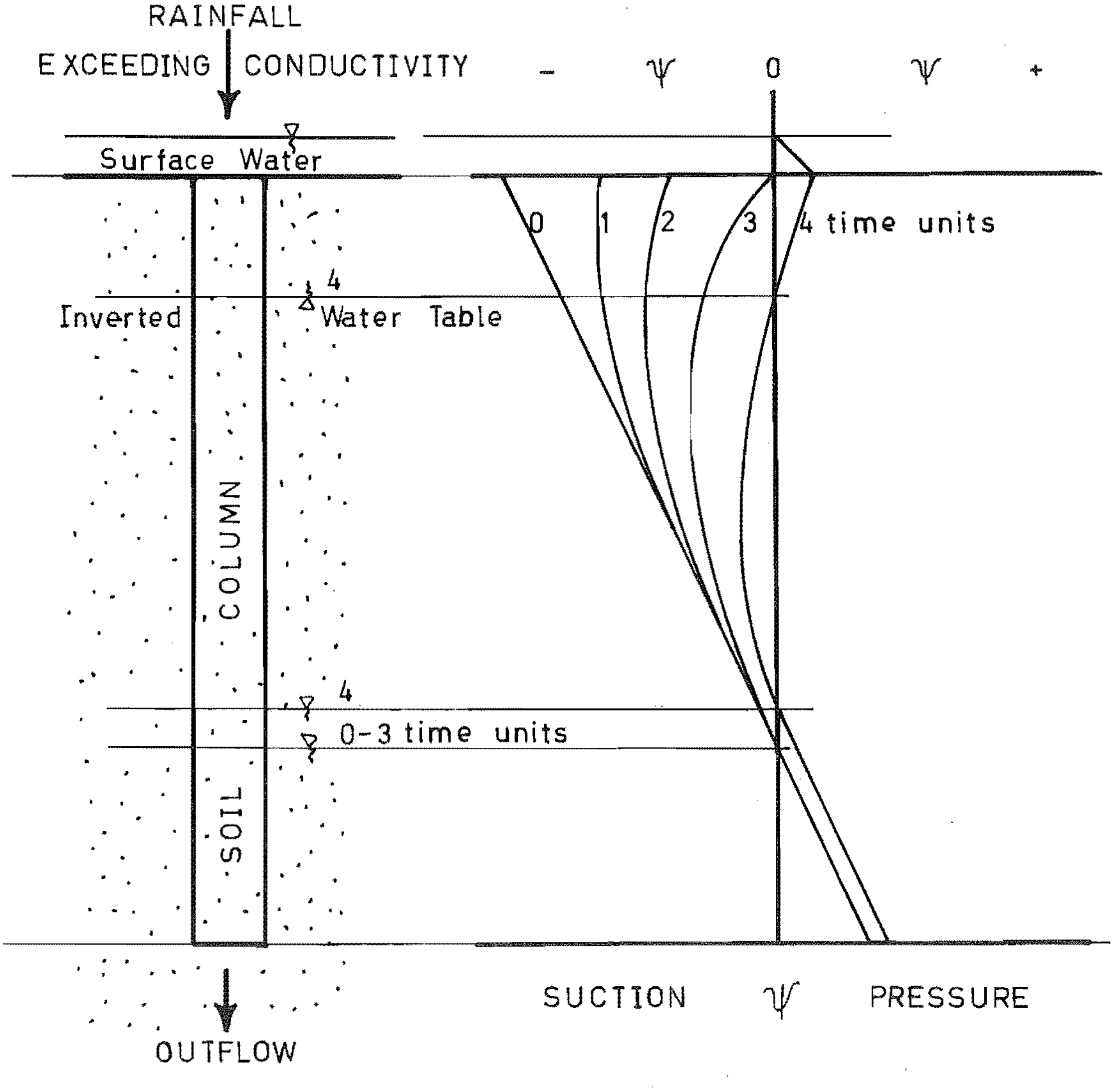


FIGURE 5-10: SATURATION FROM ABOVE

achieved between rainfall, infiltration and additions to surface water. Subroutine SOIL carries this out by iterating on the upper boundary velocity, using surface water depth as the convergence criterion. In this way the observed decrease of infiltration capacity with time is simulated.

Freeze's one-dimensional solution(28) failed to maintain this continuity during the period when a positive surface pressure was increasing to a maximum which he imposed. After this maximum had been reached his solution predicted that this pressure would propagate down into the soil as shown in Figure 5-11. However this implies a varying hydraulic gradient (and hence velocity) in the saturated region, which violates continuity. The correct behaviour is that shown in Figure 5-10 where the hydraulic gradient is constant in the saturated zone; as the inverted water table proceeds downwards the gradient of Ψ tends to, but never reaches, zero. After this inconsistency was pointed out by this writer Freeze adopted the correct method in later work(29).

Since the specific moisture capacity C is zero in the saturated zone, equation 5-2 reduces to Laplace's equation:

$$\frac{\partial^2 \Psi}{\partial z^2} = 0 \qquad (5\text{-}14)$$

which has the solution,

$$\frac{\partial \Psi}{\partial z} = \text{constant} \qquad (5\text{-}15)$$

If the entire column becomes saturated there is therefore no term involving time in the system of equations. Although equation 5-15 dictates that the gradient of Ψ and hence the

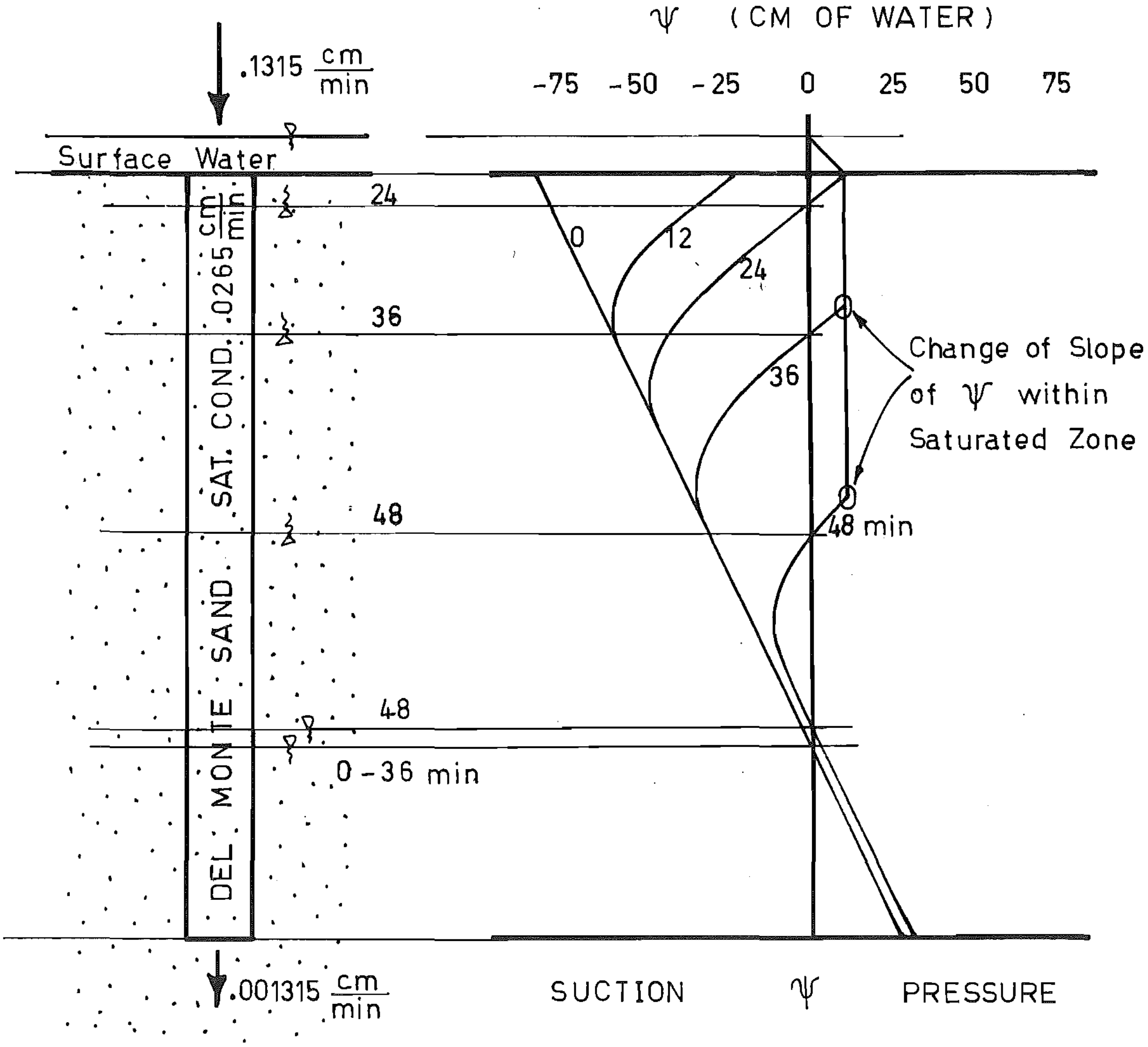


FIGURE 5-11: FREEZE'S TREATMENT OF SATURATION FROM ABOVE

velocity is constant throughout, it does not relate the solution at a given time to the conditions at the end of the previous time step. In this case the constant velocity existing throughout is assumed to be that given by the lower boundary condition, and the surplus over this of rainfall is added to the previous depth of water on the surface. This gives the updated pressure at the top of the column, from which the rest of the pressure field can be calculated via Darcy's equation.

(d) Accuracy

The accuracy of the numerical solution may be checked at any stage by applying a water balance. The net input to the soil column for any given period, as depicted by the boundary conditions, should equal the increase in moisture content integrated over the height of the column. This technique was used to trace errors arising from a variety of causes.

Initially the water balance agreement was poor, and it was suspected that this was caused by the moisture content entering into the calculation only through its derivative with respect to suction, the specific moisture capacity. Execution of the solution of the algebraic equations in double precision on the computer confirmed this suspicion by reducing this discrepancy to a few percent of the net input.

The water balance was also used to estimate the error resulting from the approximation of the differential equation by the difference scheme. This error results from the use of a backward time step, in which the distance and time derivatives are not centred on the same point, and from the assumption that the coefficients C and K are constant throughout a time step and equal to their values at the end of the previous time step. The magnitude of such errors will depend on the rate at which the values of ψ are changing with respect to time, which in turn depends on the ratio of the infiltration rate to the current unsaturated hydraulic conductivity. The higher the infiltration or the lower the conductivity the more rapidly the values of ψ will change, and the greater the error which will result. Accuracy will therefore be worst at the start of a heavy rainfall on a dry soil.

The solution for an initially dry soil column was tested with various combinations of infiltration, conductivity and time step to determine whether this error would be significant. The boundary condition at the base of the column was a velocity downwards proportional to the height of the water table in the column. The computer was programmed to print out the accumulated error after each time step, and the following information was obtained.

Most of the error occurred in the first time step. Thereafter the accumulated error increased only slightly until the top of the column became saturated (if the rainfall exceeded the conductivity) and then decreased until the column became saturated throughout.

The initial error sometimes exceeded the volume of rainfall applied during the first time step. The final accumulated error following complete saturation was always less than this amount. No further error occurred following complete saturation because the coefficients C and K are constant.

The size of the time step made relatively little difference to the initial error. Applying a rainfall of 10 inches per hour to a soil of saturated conductivity of 5 inches per hour, the initial error of 1.5 inches using a 1-hour time step was decreased only to 0.4 inches after decreasing the time step by a factor of 128.

Thus the non-iterative scheme described in (b) above would not be suitable for describing the immediate effects of a sudden large change in boundary condition. But because many time steps would be used to describe a storm rainfall it was felt that the scheme as described would be satisfactory for its intended use in a catchment model. Moreover early tests with the model indicated that hydraulic conductivity values would have to be high compared to rainfall intensities, further lessening the problem.

Additionally the water balance technique enabled the optimum size of the distance increment to be found. The distance increment used in equations 5-7, 5-8 and 5-9 to calculate the distance derivatives was chosen for simplicity to be constant over the depth of the column. Its size is dictated by the zone in which ψ changes most rapidly with depth, that is, the vicinity of the surface. A distance increment too large will prevent accurate depiction of the distance derivatives of equations 5-2 and 5-3; equation 5-3 is especially important since it provides the means to introduce the rainfall-dependent upper boundary condition.

The effect of varying the distance increment was examined after the soil column solution had been incorporated into the catchment model. The accuracy was monitored on a simulation of a real storm for which both rainfall and riverflow data were available. Besides determining the accumulated error for the whole storm, any other change in the behaviour of the soil solution could be observed as a change in the coefficient of variation, a measure of the agreement between the model output and the recorded riverflow.

The accumulated error (see Figure 5-12) closely approached the final value (which was 6% of the storm rainfall volume) with only five depth increments describing a 25-inch soil column. The remaining error was assumed to be due to the finite time step used. The soil solution itself, as measured by the coefficient of variation, did not stop changing until the number of depth increments reached 25. This number of increments was therefore used for all further simulations.

C. Boundary and Initial Conditions

Having discussed the numerical details of the soil column solution above, the actual boundary and initial conditions which enable the solution to fit into the AM are now given.

The boundary condition at the upper end of the soil column is a velocity downwards equal to the rainfall rate for the current time step, minus the rate of interception and evaporation as calculated in Sections 5.2 and 5.3. If the solution predicts saturation at the top, implying that there exists water on the land surface, this velocity is reduced until continuity is attained between the net rainfall, the upper boundary velocity and the increase in surface water.

The output from the lower end of the column represents subsurface flow. If the column is to model the effect of a catchment in translating rainfall into riverflow, the soil column must exhibit a similar damping and delaying relation

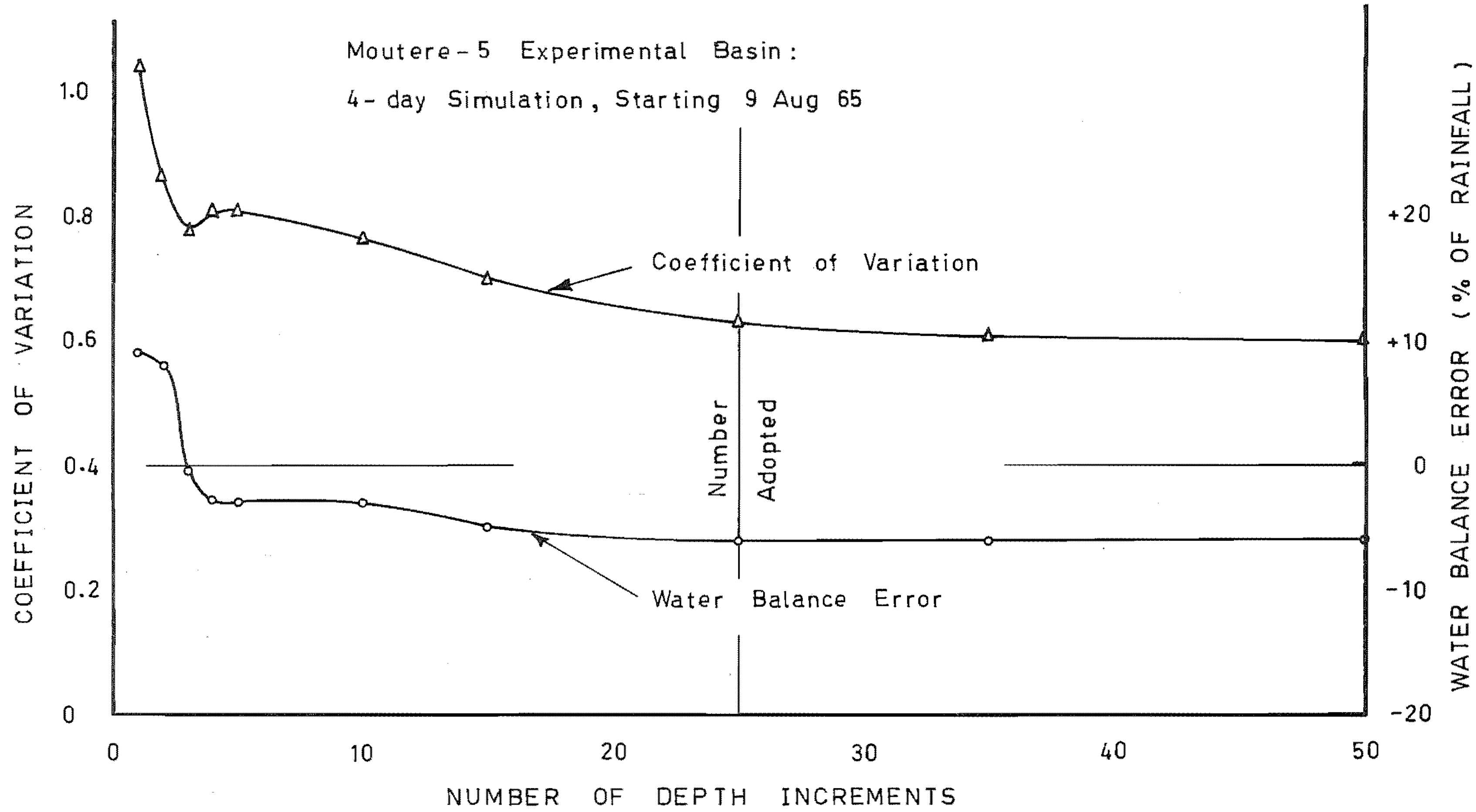


FIGURE 5-12: VARIATION IN COLUMN SOLUTION ACCURACY WITH DEPTH INCREMENT SIZE

between input (at the upper end), and output (at the lower end). This is achieved by making the lower boundary condition a downward velocity dependent on the moisture conditions within the column, as depicted by the height of the water table found by the previous time step. This velocity varies linearly with the water table height because this gives a linear semilog recession in the absence of rainfall. The velocity is given by:

$$v_{LB} = \text{constant} \cdot \frac{h_{WT}}{h_{COL}} \cdot K_{SAT,V} \qquad (5\text{-}16)$$

where h_{WT} is the height of the water table in the column,
h_{COL} is the height of the soil column, and
$K_{SAT,V}$ is the saturated conductivity in the vertical direction

The constant in equation 5-16 may be estimated by interpreting the soil column base outflow, integrated over a catchment slice, as the lateral outflow from such a slice, both cases being fully saturated ($h_{WT}=h_{COL}$). Referring to Figure 5-13, we have the sum of all column base outflows from equation 5-16:

$$q_V = \text{constant} \cdot K_{SAT,V} \cdot L \qquad (5\text{-}17)$$

where q_V is the sum of column outflows per unit width of slice, and
L is the length of the slice

Similarly we have the lateral flow, this time using Darcy's equation:

$$q_H = K_{SAT,H} \cdot S \cdot H \qquad (5\text{-}18)$$

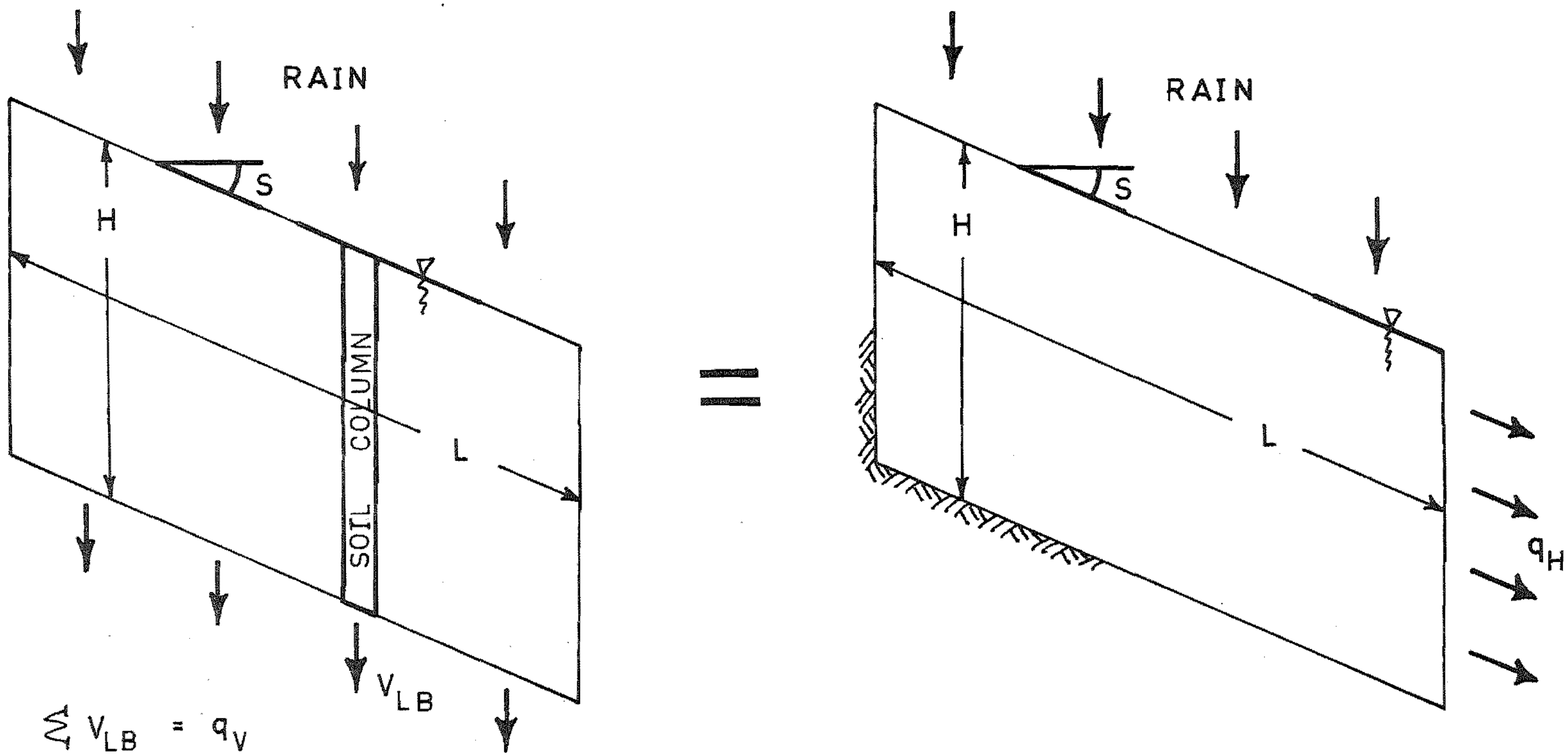


FIGURE 5-13: DEFINITION SKETCH FOR THE SOIL COLUMN OUTFLOW PROPORTIONALITY CONSTANT CALCULATION

where q_H is the lateral flow per unit width of slice,

$K_{SAT,H}$ is the saturated conductivity in the horizontal direction,

S is the surface slope, and

H is the slice depth.

Equating q_V and q_H gives:

$$\text{constant} = \frac{K_{SAT,H}}{K_{SAT,V}} \cdot \frac{H}{L} \cdot S \tag{5-19}$$

Considering the terms on the right-hand-side of equation 5-19, the ratio of saturated conductivities is the only one likely to be greater than one; the value would probably lie in the range one to ten. The ratio of subsurface flow depth to length on the other hand will be many orders of magnitude less than one, while the average surface slope will also be less than one. The value of this constant will therefore be very much less than one. The consequence of this is that the maximum value of v_{LB}, when the column is fully saturated, will be by equation 5-16 very much less than $K_{SAT,V}$. This will be referred to when discussing the input-output behaviour of the column.

The above argument assumes that flow can occur steadily as shown in Figure 5-13 (b). Since a constant water surface slope (and hence velocity) combined with constant depth implies no rainfall input, this situation violates continuity. However the average water surface slope must lie between $S+\frac{H}{L}$ and $S-\frac{H}{L}$; and therefore, since $\frac{H}{L}$ is small compared to S, a valid water surface profile would not alter the order-of-magnitude analysis above.

Initial conditions are specified by one of three methods:

(a) A realistic array of pressure or suction Ψ, throughout the soil depth.

(b) A soil moisture deficit, being the volume of water required to saturate the column.

(c) The water table height as a fraction of the height of the column. In cases (b) and (c), an equivalent array of Ψ is calculated assuming zero flow before the solution is commenced.

As in the SWM a constant fraction of the subsurface flow, as given by the lower boundary velocity, is assumed to bypass the catchment outlet. The remainder is added to the surface flow to become the riverflow for the current time step.

D. Soil Properties

The relations between hydraulic conductivity, moisture content and soil suction, illustrated in Figures 4-1 and 4-3, may not be known for a real catchment, particularly for a heterogeneous one; in this case representative relations must be found by trial-and-error fitting of model output to recorded riverflows. For this reason the relations are assumed constant throughout the column, and are described in the model by a three-parameter equation of the form:

$$\theta \text{ or } K = \frac{90}{C}\left\{1 + .636 \tan^{-1}\left(\frac{\Psi + A}{B}\right)\right\} \qquad (5\text{-}20)$$

where θ is the moisture content,

K is the hydraulic conductivity,

A and B are constants with the dimensions of Ψ, and

C is a constant with the dimensions of $(\theta \text{ or } K)^{-1}$

This assumed form is based on curves presented by Freeze[28] for three different soils. All six curves have a similar shape and can be approximated by equation 5-20. The values of A and B determine the shape of the curve and are therefore constant from one soil to another. The value of C scales the curve so that it adopts values appropriate to the soil under consideration. In this way only two parameters (the values of C for the two relations) need to be determined by fitting model output to riverflow. The standard values of A and B and the range of values of C to be expected are given in Table 5-1.

If more detailed knowledge of the soil properties is available, the information may be input via a table, or by choosing different values of A and B as well as of C. Inclusion of hysteresis between wetting and drying is not possible in the present model. It is considered that for peak prediction the wetting curves would be more appropriate and these were used to derive the quoted values of A, B and C in Table 5-1.

Table 5-1: Values for the Constants Describing the Hydraulic Conductivity and Moisture Content Variation

MOISTURE CONTENT-SOIL SUCTION	
A=13.8 inches of water	B=27.6 inches of water
C=11,600 - Saturated Moisture Content=.01	
C=116 - Saturated Moisture Content=1.0	
HYDRAULIC CONDUCTIVITY-SOIL SUCTION	
A=19.7 inches of water	B=7.88 inches of water
C=1600 $(in/hr)^{-1}$ - Saturated Conductivity=0.1 in/hr	
C=0.4 $(in/hr)^{-1}$ - Saturated Conductivity=400 in/hr	

E. Input-output Behaviour

The behaviour of the soil column solution is illustrated below by several examples which have been synthesised from the results of actual model simulations. In each case the response was obtained by subjecting the model to an artificial storm, while suppressing by parameter choice the action of all model components other than the subsurface component. Surface flow was prevented by choosing a zero slope and high roughness, and interception was prevented by setting the interception capacity to zero. The initial conditions were a water table at the bottom of the column and hence no initial lower boundary outflow, and a distribution of ψ appropriate for zero flow.

Figure 5-14 shows the response of the column to a rainfall of finite duration and constant intensity greater than the saturated hydraulic conductivity, falling onto a dry soil. The response exhibits the following features in chronological order:

(a) The surface becomes saturated at time T_1. The greater the ratio of rainfall intensity to saturated conductivity, the smaller the value of T_1. This is consistent with the standard infiltrometer test in which water is supplied at whatever rate is necessary to saturate the surface; this rate is initially very large for a dry soil and the time T_1 is practically zero.

(b) After time T_1 the upper boundary velocity (infiltration) falls below the rainfall rate, as the model maintains continuity between the rainfall, infiltration and increases to surface water. The decrease in infiltration decays with time, the infiltration rate tending to the

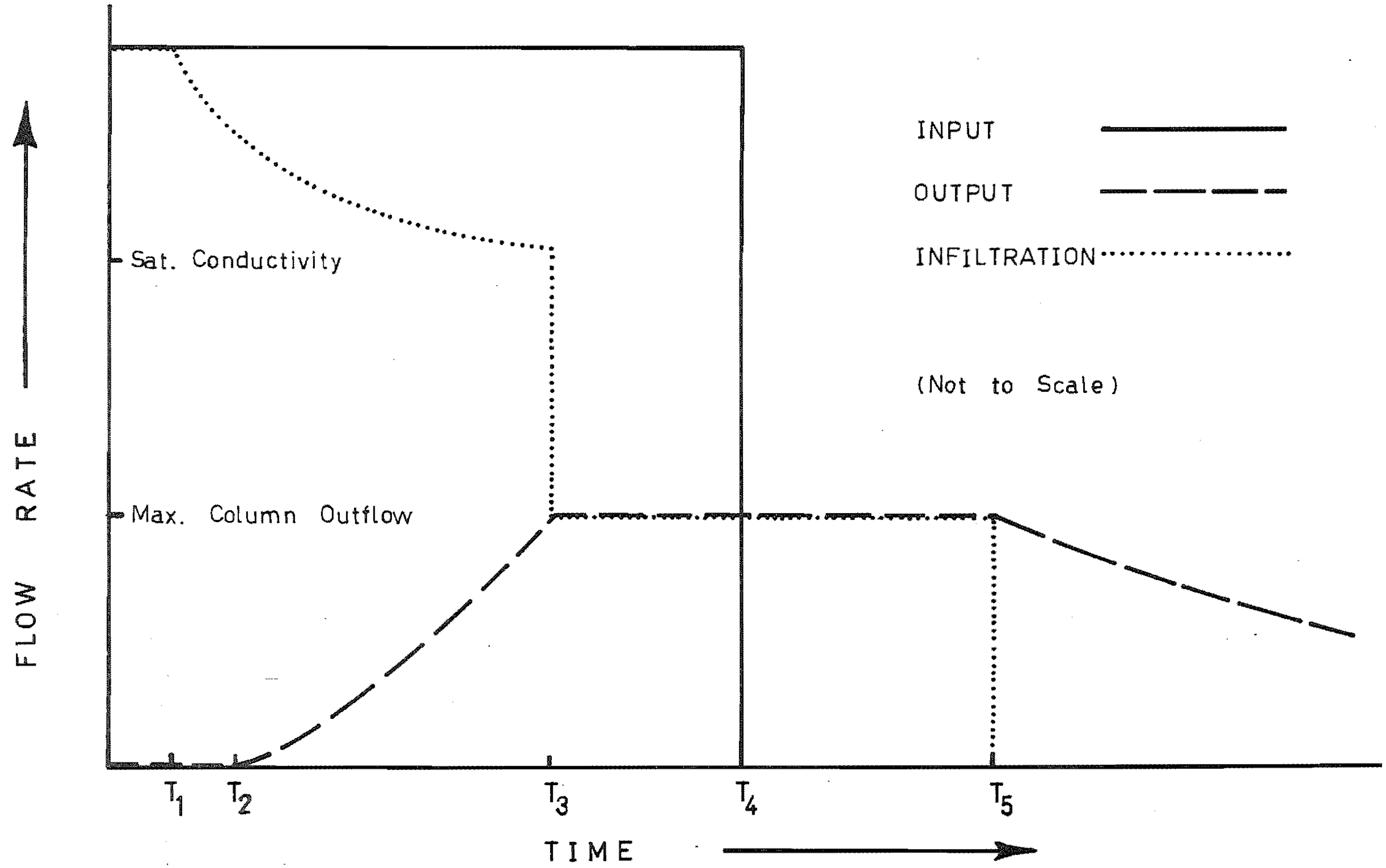


FIGURE 5-14: INPUT-OUTPUT BEHAVIOUR OF THE SOIL COLUMN SUBJECT TO A RAINFALL EXCEEDING THE SATURATED CONDUCTIVITY

saturated hydraulic conductivity. This duplicates the familiar shape of the infiltrometer curve which shows a decrease in infiltration capacity with time.

(c) The region influenced by the infiltrating water proceeds deeper with time, eventually reaching the bottom of the column at time T_2. This causes the water table, initially at the bottom of the column, to attempt to rise to such a level that the column outflow given by equation 5-16 balances the rate of water arrival at the water table. This situation corresponds to the lag observed between rainfall events and rises in groundwater levels, dependent on the hydraulic properties of the soil and the depth to the water table.

(d) Since the lower boundary velocity, even with a fully saturated column, is much less than the saturated conductivity (see Part C above), the lower boundary outflow will not be able to balance the rate of inflow to the saturated zone, and the water table will eventually reach the top of the column at time T_3. When this happens the infiltration reduces immediately to the maximum value of the lower boundary velocity (given by equation 5-16) to satisfy continuity requirements. This abrupt drop in infiltration is a consequence of the one-dimensional nature of the column; it is not observed in infiltrometer tests because lateral flow cannot be prevented, especially at large times from the start of the test, nor is it observed in real catchments because the time T_3 will be different for different parts of the catchment. In this respect the solution represents a gradual reduction in infiltration by a step function.

(e) After the column saturates the excess of rainfall over the maximum lower boundary outflow adds to water on the surface. If at time T_4 the rain stops infiltration at the lower boundary outflow rate depletes the surface water until the surface becomes unsaturated at time T_5. The response of the model in this period will depend on the operation of the surface flow component.

(f) After time T_5 the water table, and hence the column output, declines because of the outflow from the bottom of the column. The linear dependence of the outflow on the water table height imposes a recession that is linear on a semilog plot, in accordance with observed riverflow recessions.

The behaviour for a rainfall which is less than the conductivity but greater than the maximum column outflow is similar to that described above, except that infiltration continues at the rainfall rate until time T_3, when the rising water table reaches the surface.

The behaviour for a rainfall which is less than the maximum column outflow consists of a water table rise to the level where the bottom outflow can transmit the infiltration, which must be equal to the rainfall. The outflow therefore rises to approach the input (see Figure 5-15), although in practice the steady state would rarely be reached because of unsteady rainfall rates.

The recession of column outflow from a fully-saturated condition is shown on a semilog graph in Figure 5-16. Also shown is the recession subject to a constant evaporation, which hastens the recession as would be expected. The departure from a straight line at the end of the recession

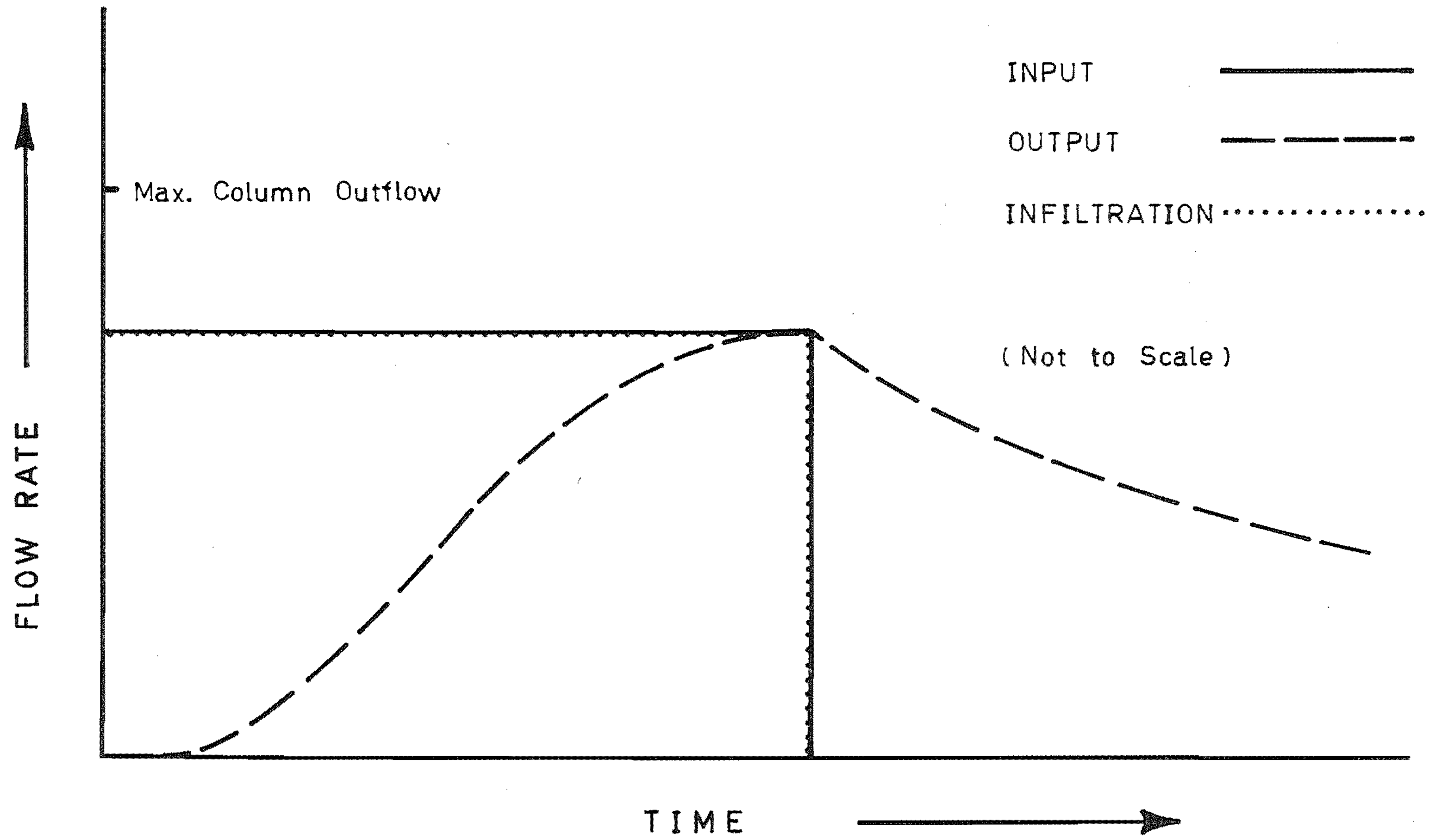


FIGURE 5-15: INPUT-OUTPUT BEHAVIOUR OF THE SOIL COLUMN SUBJECT TO A RAINFALL LESS THAN THE MAXIMUM COLUMN OUTFLOW

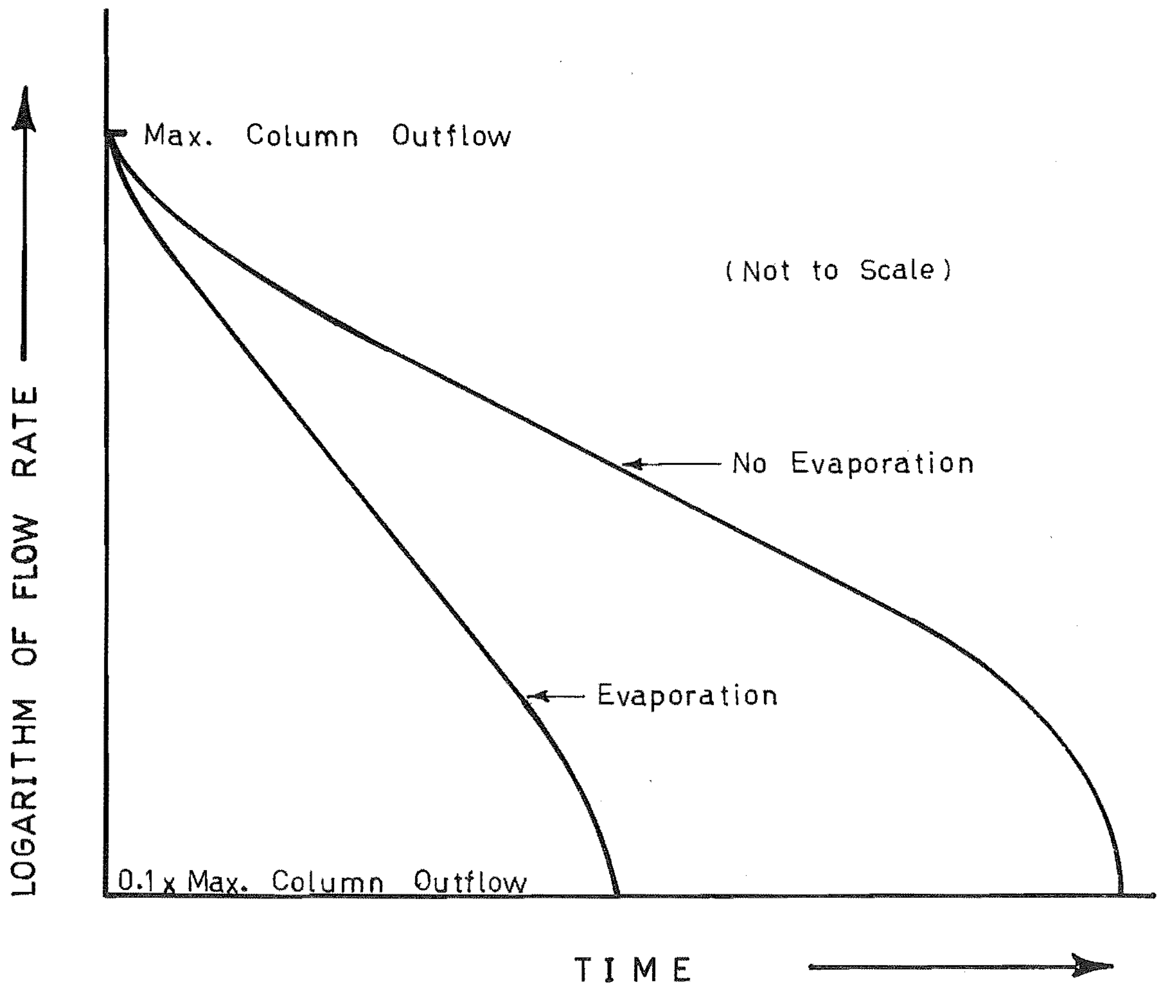


FIGURE 5-16: RECESSION BEHAVIOUR OF THE SOIL COLUMN

results from the finite-difference method of solution which is incapable of allowing the flow to approach zero asymptotically. When the water reaches one or two distance increments above the bottom of the column the solution predicts a rapid desaturation which brings the flow to zero. This is not a fault; all ephemeral streams exhibit this departure from semilog linearity when they dry up.

It is seen that the soil column solution reproduces the important features of the behaviour of a homogeneous soil - the infiltration capacity curve, the dependence of the soil moisture and groundwater level on the rainfall, saturation of the surface and the decline of the water table when the rainfall ceases. Success of the component in the model will therefore depend on how well average values of the soil properties can match it to a heterogeneous catchment.

F. Assumptions

In addition to the assumptions made in deriving Richards' equation in Section 4.2, the following assumptions have been made in applying the one-dimensional form of the equation to the catchment situation:

(a) The one-dimensional soil column solution can, by suitable choice of properties, be made to have similar input-output behaviour to the (three-dimensional) subsurface zone of a catchment (as shown in Section 5.4D).

(b) A vertically-downward outflow from the bottom end of a column of constant hydraulic properties can represent the lateral subsurface flow caused by the typical decrease of hydraulic conductivity with depth.

(c) The wetting limbs of the hysteretic relations between hydraulic conductivity, moisture content and soil suction give an adequate representation for modelling peak flows.

(d) The relations between the hydraulic conductivity, moisture content and soil suction maintain the same shape for different soils, and that shape is given by equation 5-20.

5.5 Surface Flow

Surface flow is modelled in the AM as in the SWM by the equations of motion for flow over a plane of constant slope with lateral inflow, modified by Crawford and Linsley so that input and output can be expressed as depths of water over the catchment, as required for use in a lumped model. In the AM this component operates, whenever the soil column solution predicts saturation of the land surface, by using the current depth of water on the surface and the current rate of increase of surface water to predict the flow into the river from the lower end of the plane representing the catchment surface. Full details of this component are given by Crawford and Linsley[17]. They compared its behaviour with the experimental measurements of Izzard[40] and others, and found satisfactory agreement. The input-output behaviour obtained from the AM response to a constant rainfall, with soil properties chosen to prevent infiltration, is illustrated in Figure 5-17.

If any potential evapotranspiration remains unsatisfied after removing water from the interception storage, this is allowed to remove water from any surface water on the soil. Any potential evapotranspiration remaining after this is subject to reduction by Boughton's method (see Section 5.3)

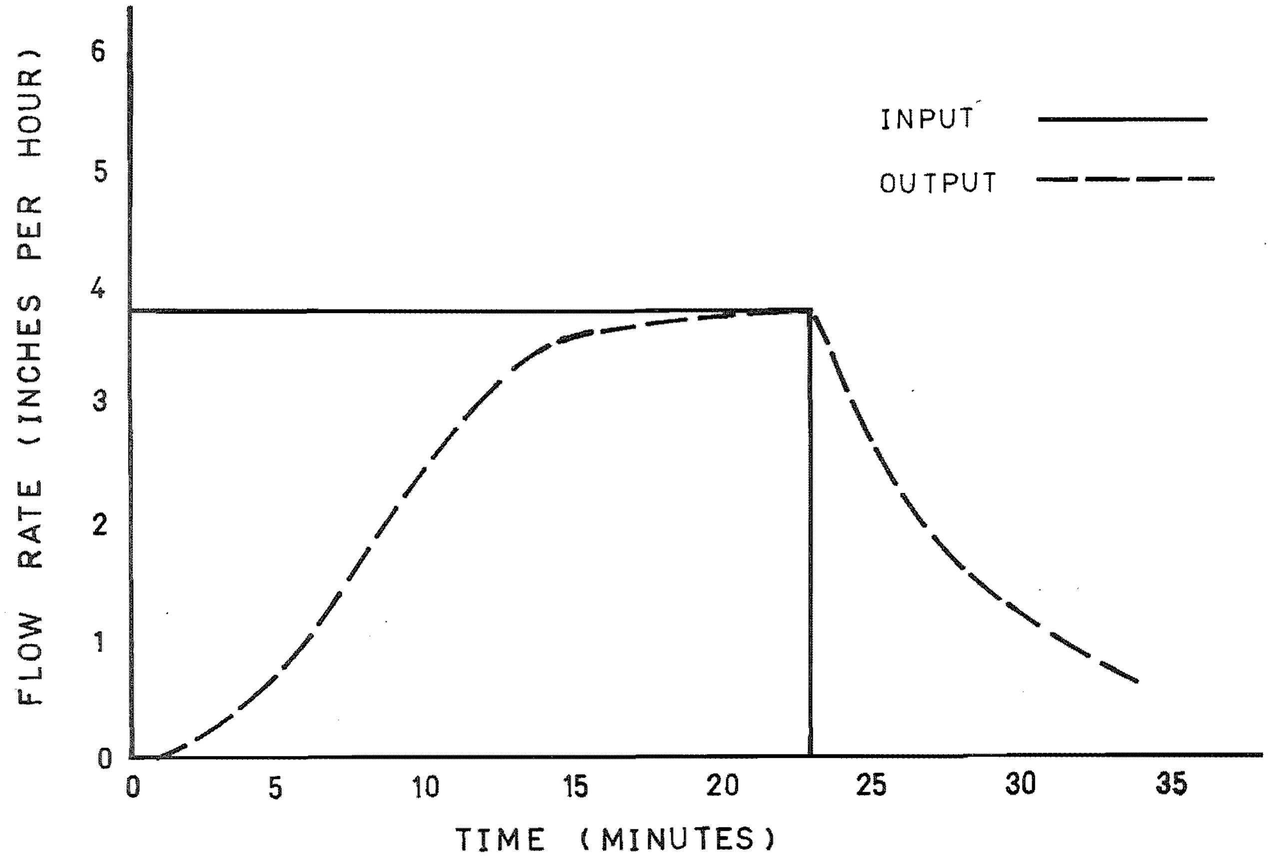


FIGURE 5-17: INPUT-OUTPUT BEHAVIOUR OF THE SURFACE FLOW COMPONENT

before being applied as the upper boundary condition to the soil column solution.

Surface flow and evaporation from surface water change the conditions at the top of the soil column, making necessary some adjustment within the column. It is not possible merely to subtract the flow and evaporation from the current surface water depth since a change in the surface depth would have an effect throughout the column. This adjustment is made by treating the surface flow and evaporation calculated at one time step as an abstraction from the soil column upper boundary condition for the next time step. As well as keeping the surface and subsurface solutions compatible at their common boundary this satisfies continuity between rainfall, infiltration and surface water, flow and evaporation. The adjustment is carried out thus:

(a) Soil calculations at time step t first predict a positive pressure at the surface. The upper boundary velocity is adjusted and the soil column solution recalculated until the rainfall volume equals the infiltration plus the increase in water on the surface (see Part C above).

(b) At time step t+1, evaporation is allowed to occur from surface water and a surface flow calculation is made based on the water depth and depth increase rate from the last time step.

(c) The depth of water removed from the surface in (b) is <u>subtracted</u> from the net rainfall (depleted by interception) to deduce the upper boundary velocity to be used for the soil column solution for time step t+1.

(d) The soil column solution is carried out for time step t+1, and the process is repeated.

CHAPTER SIX

TESTING AND PERFORMANCE

6.1 Testing Program

In seeking to determine whether a description of the subsurface zone of a catchment by Richards' equation will improve the performance of a catchment model we have replaced the subsurface components of the SWM by the Richards' equation solution for a vertical column of soil. We now need to test this AM on some real catchments in order to evaluate the effect of the amendments.

Three catchments suitable for this test are chosen and described. A method for calculating the time interval needed to adequately represent peak riverflow values is presented and used to find the time intervals required for each of the three catchments. After examination of possible alternative methods for evaluating those model parameters not able to be measured it was decided to adopt a manual adjustment technique, using a graphical sensitivity analysis and simultaneous plotting of both recorded and simulated riverflows. Using 16 of the 26 storm periods available, the parameters for the three catchments for both the SWM and the AM are found using this technique. After this test of fitting ability, both the SWM and the AM are tested for prediction ability on the remainder of the periods. Finally the performance of the AM is discussed and compared with that of the SWM.

6.2 Data

A. Catchment Choice

Several catchments were required from which to obtain rainfall, potential evapotranspiration and riverflow data to test the AM. The catchments had to satisfy the following criteria:

(a) Size. The catchments had to be hydrologically small since the AM does not consider the effect of channel flow on the rainfall-riverflow process.

(b) Data Availability. Records of rainfall and riverflow at sufficiently small time intervals to adequately define the peaks were needed; daily evaporimeter records were also desirable.

(c) Catchment Variety. A range of conditions including size, infiltration characteristics, vegetation and rainfall was desirable.

Three catchments in New Zealand which best meet these criteria have been selected. Their locations are shown in Figure 6-1, and their important properties summarised in Table 6-1.

Suitable records at small time intervals are scarce in New Zealand so some of the criteria have not been fully met. All three catchments are hydrologically small, although Moutere only just meets the criterion of negligible channel effects. The amount of data able to be collected for each catchment was restricted by the recent (1968) instrumentation of Reynolds, the recent use of the Moutere River for summertime irrigation supply and the labour required to extract rainfall volumes from raingauge charts. Flows were more readily available, being on computer cards or as computer printout from the operating agency, the New Zealand

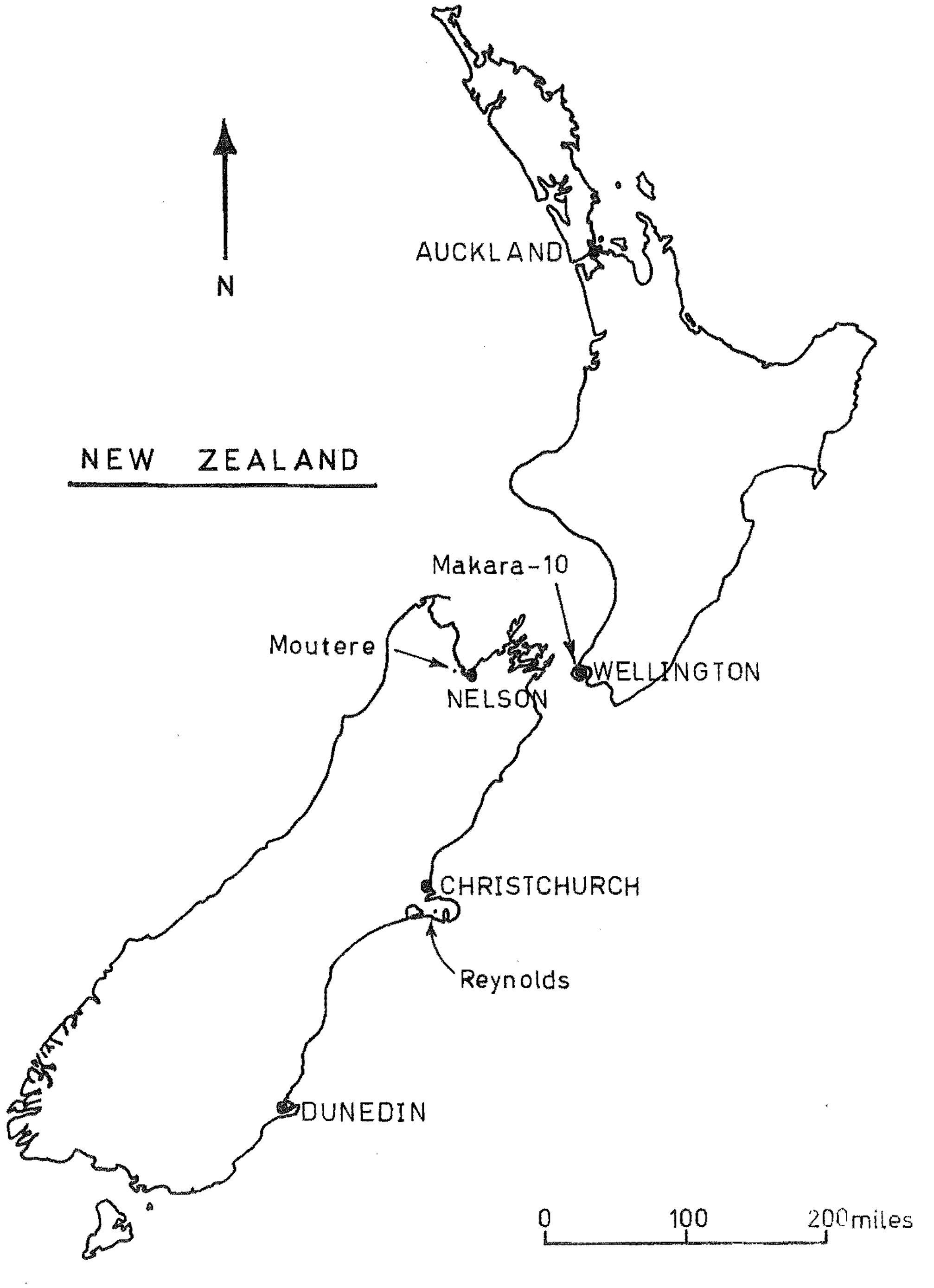


FIGURE 6-1: LOCATION OF THE CATCHMENTS USED

Table 6-1: Summary of Catchment Properties

CATCHMENT	Makara-10	Reynolds	Moutere
LOCATION	Wellington	Banks Peninsula	Nelson
SIZE	14ac(5.6ha)	1.2mi^2(3.2km^2)	24mi^2(61km^2)
SLOPE	.58	.40	.10
ANNUAL RAINFALL	43in(1090mm)	38in(970mm)	45in(1140mm)
VEGETATION	Open Pasture	Grass at Lower Levels, Tussock Higher	Pasture, Pine Forest, Intensive Cultivation
SOIL	Central Yellow-Brown Soils and Stony Loam	Yellow-Grey Soils and Brown Granular Loam	Central Yellow-Brown Soils
GEOLOGY	DeeplyWeathered Greywacke	Basalt and Andesite Flows	Gravels

Ministry of Works. The variation between the catchments of size, slope, vegetation and geology is good, but the soils and annual rainfalls are similar.

The next part of this section contains a general description of the three catchments. The subsurface description is then given under a separate heading, because of the emphasis on the subsurface processes in this study. Lastly the translation of the rainfall, evapotranspiration and riverflow data into a form suitable for input to the models is described.

B. General Description

(a) Makara-10

This catchment is one of a group of small, experimental catchments established by the Water and Soil Division of the Ministry of Works[41] to study the effects of land management techniques on the hydrological characteristics. During the period from which the records were obtained (1960-1965) the catchment, which consists of unimproved pasture, has been hard grazed by sheep and cattle. The only treatment during this time was the clearing of infestation by Cassinia (an evergreen shrub) in 1963 and 1964.

Makara is 12 miles (19km) west of Wellington city. Catchment 10 (see Figure 6-2) covers 14 acres (5.6ha), rising from 150 to 600 feet (45 to 180m) above sea level. The catchment is steep (.58feet/foot), with little flat area. The channel slopes at .32feet/foot in an incised bed and the flow is perennial.

A Dines tilting-siphon automatic raingauge is situated near the top of the catchment while four five-inch daily manual gauges are distributed around the perimeter. Flow measurement is carried out in a three-foot H-flume with a daily Kent recorder. Flows can be read at intervals down to two minutes but the raingauge chart scale allows reading only to ten minutes

(b) Reynolds

Ninety hydrological regions have been delineated for New Zealand[42] and representative basins set up in most of them. The Reynolds catchment is one of these, instrumented in 1968, and situated on Banks Peninsula 25 miles (40km) east of Christchurch city. The peninsula consists of former volcanic craters into which the sea has

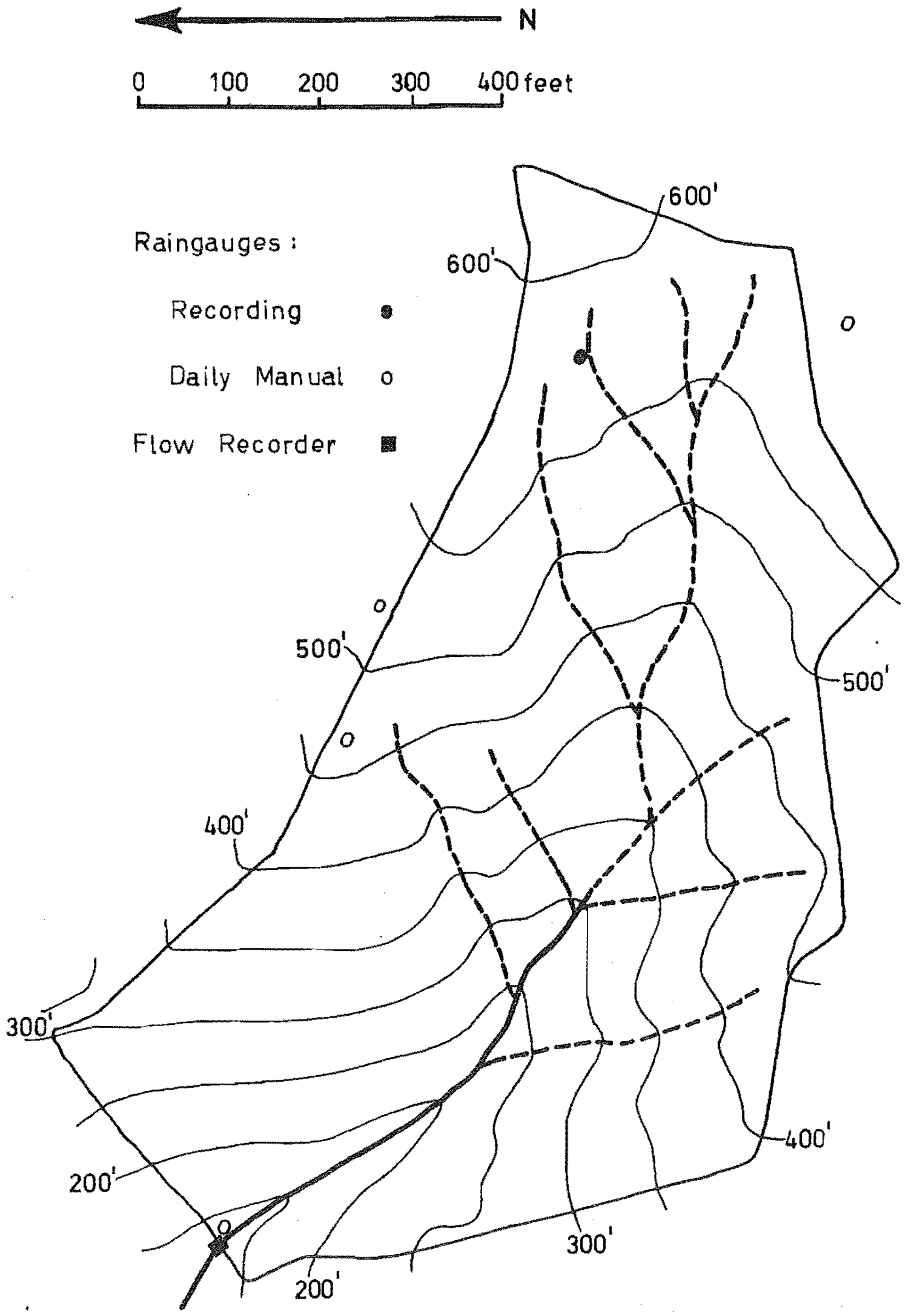


FIGURE 6-2: THE MAKARA-10 CATCHMENT

intruded to form the Lyttelton and Akaroa harbours.

Reynolds catchment (see Figure 6-3) is a steep (.40feet/foot), fan-shaped area, covered by exotic grasses with tussock at higher levels and remnants of native bush near the main channel. The area is 1.2 square miles (3.2km^2) and the altitude ranges from 400 to 2200 feet (120 to 670m). Flow in the Reynolds Stream, which slopes at approximately .20feet/foot, is perennial.

A Lambrecht automatic raingauge is located near the catchment outlet; five storage and one daily manual raingauge are distributed fairly uniformly over the catchment. The Lambrecht chart can be read to ten minutes. A digital water-level recorder which punches a signal every fifteen minutes monitors the stream stage behind a concrete shallow-vee weir. The weir has been rated only to about 10% of the maximum recorded flow, but the area-velocity extrapolation of the stage-discharge curve agrees with weir-flow calculations.

(c) Moutere

Moutere catchment is also a representative basin. It has been operated by the Ministry of Works in conjunction with the Nelson Catchment Board since 1961, but extensive summertime irrigation withdrawal has been carried out since 1969; the lack of information on the removals jeopardises the future of Moutere as a representative basin. The area, which is 12 miles (19km) west of Nelson city, consists of gently rolling country underlain by the Moutere gravel formation.

The Moutere catchment (see Figure 6-4) is mainly flat, with some low hills in the head of the basin; the altitude range is 100 to 1100 feet (30 to 330m). The area of 24

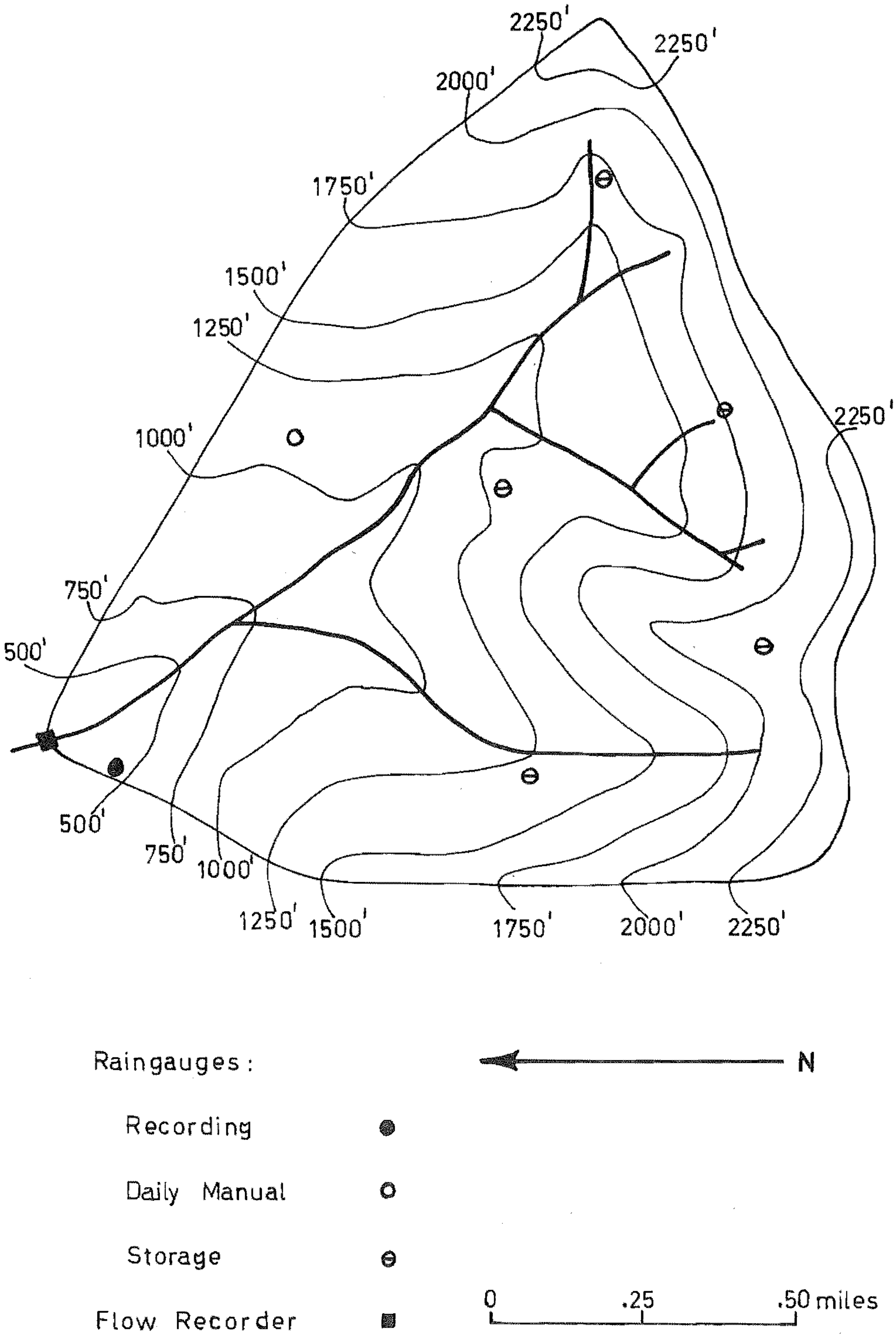


FIGURE 6-3: THE REYNOLDS CATCHMENT

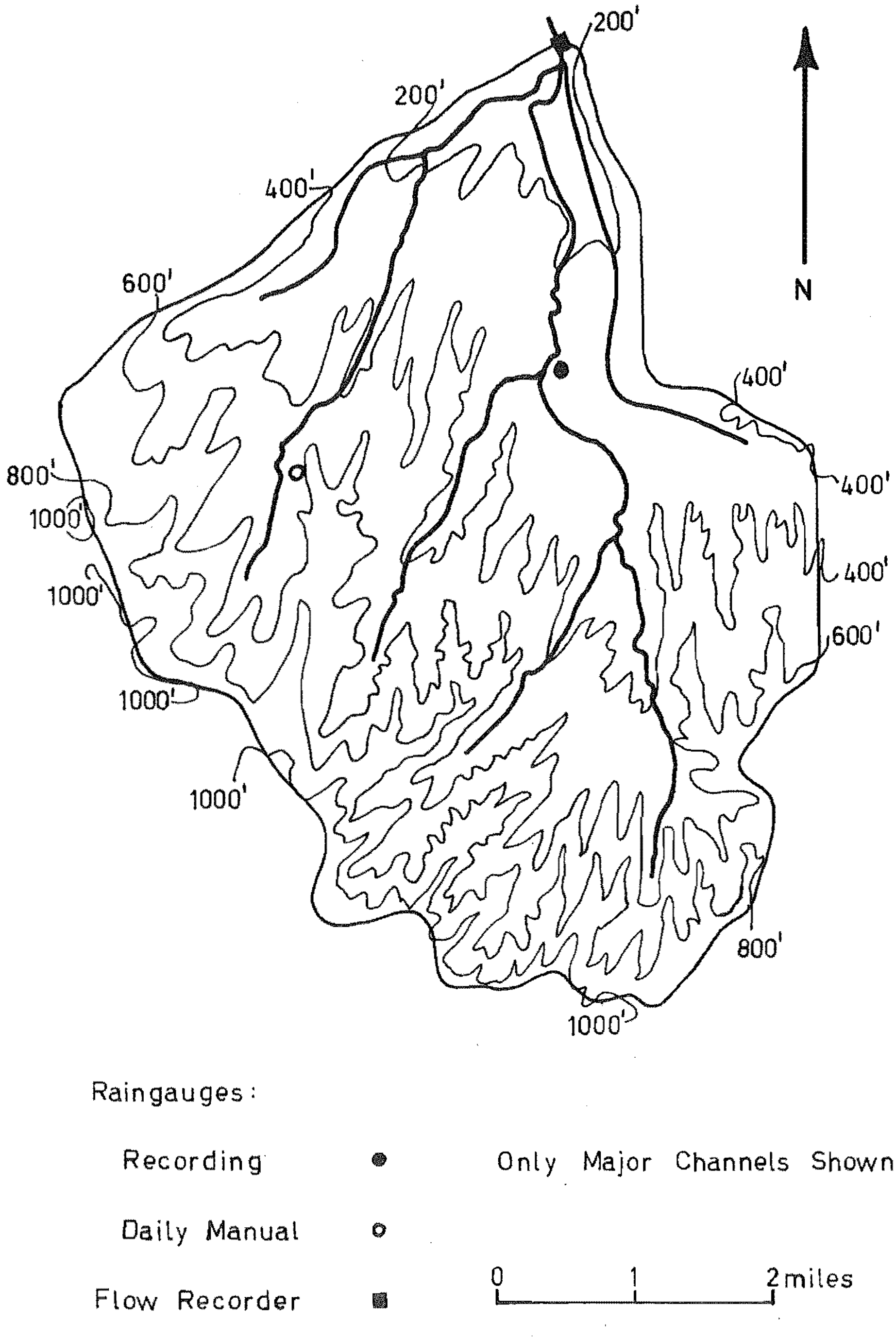


FIGURE 6-4: THE MOUTERE CATCHMENT

square miles ($61km^2$) consists of exotic grasses (grazing), pine forest (about 20% by area) and intensive cultivation (fruit, tobacco and hops) at the lower levels. The average slope of the perennial channel is .02 feet/foot.

A Casella weekly automatic raingauge operated until 1969, when a Lambrecht gauge was installed. The Casella charts could be read to 60 minutes, the Lambrecht charts to ten minutes. Two daily and three storage gauges were originally installed, but after obtaining catchment-mean-rainfall correlations with the automatic gauge, all of these except one daily manual gauge were removed. A digital water-level recorder operating on a fifteen-minute time interval records the stage behind a concrete broad-crested weir, which has been rated up to the maximum recorded stage. Records were chosen from the 1967-1969 period.

C. Subsurface Description

Since the model changes emphasise processes which occur below the land surface a separate description of the subsurface zone of each catchment is given. No measurements of the unsaturated hydraulic properties of the soils (the relations between the hydraulic conductivity, moisture content and soil suction) are available for these catchments or for any soils in New Zealand except for those associated with agricultural studies in some areas. Similarly no extensive infiltrometer testing has been reported in New Zealand, although this information if available would be only indirectly relevant. Hence the only data available are the descriptions of the soil type and the subsoil conditions as given by the Ministry of Works[41,42]. The collection of more extensive soil data is a matter of urgency.

(a) Makara-10

The soils are Central yellow-brown earths and related steepland soils. 98% consists of Makara stony loam, and 2% of Korokoro silt loam. The parent material is deeply weathered Greywacke.

Subsurface flow is therefore expected to occur to a moderate depth, with some loss via the weathered parent material.

(b) Reynolds

Reynolds soil consists of brown granular loams with some yellow-grey earths. The catchment is underlain by basalt and andesite flows which occasionally outcrop.

Observation of cuttings on the catchment indicates that the subsurface flow zone will be shallow. The nature of the underlying rock suggests that little subsurface flow will bypass the outlet.

(c) Moutere

Moutere soil consists of Central yellow-brown earths derived from the Moutere gravel formation which underlies it.

This information does not allow a guess at the depth involved in subsurface flow. Scarf(43) states that the gravel formation is essentially impermeable so flow bypassing the outlet is expected to be small.

D. Data Extraction

A total of 26 storm periods, some containing several distinct peaks of flow, was selected for testing the AM. After conversion to the form of volumes in successive time intervals the rainfall, evapotranspiration and riverflow data were punched onto computer cards for input into the computer program which carries out the calculations comprising the model. The following prior processing was needed:

(a) Rainfall

Rainfall from the single automatic gauge in each catchment was read from the charts as a series of volumes in successive time increments. The Reynolds and Moutere figures were then weighted to agree with the catchment mean daily rainfalls as calculated by the Thiessen method. (This is equivalent to assigning to each daily manual or storage gauge the time pattern of the automatic gauge.) The figures for Makara-10 were left unweighted because comparison of daily mean rainfalls with daily automatic gauge totals showed no significant difference.

(b) Evapotranspiration

No pan evaporation data were available for the periods selected for any of the catchments. Potential evapotranspiration figures were based instead on estimates of monthly evapotranspiration calculated by the Penman method. Daily figures as required for the model input were obtained by dividing the monthly totals by the number of days in the month. Values for Makara-10 for the selected periods were supplied by the Ministry of Works, but these were not directly available for Reynolds or Moutere. For Reynolds, the monthly mean Penman values for the Selwyn catchment, 50 miles (80km) away, were the closest that could be obtained and these were used with reservation. For Moutere, Penman calculations had been made but not for the period in which the storms occurred, so average monthly values from other years were used.

Considerable crudity is evident in this data which would definitely not suffice for long-term simulation. However, for peak simulation over short periods the contribution of evaporation is known to be small.

(c) Riverflows

Values of riverflow for representative and experimental catchments are collected by the Ministry of Works as part of the representative basin program. These flows could be obtained either as computer listings of flows at various intervals depending on the flow rate (Makara-10), or directly as mean flows for regular time intervals on computer cards (Reynolds and Moutere). Processing involved reduction to regular time intervals where necessary, and conversion to units of depth over the catchment (volume) in successive time intervals.

The storm periods isolated are described in Table 6-2. Those periods in "Group 1" are the ones used for parameter derivation while those in "Group 2" were used for testing the prediction ability of the models. There were insufficient storms to form a "Group 2" for Moutere, but the "Group 1" storms on that catchment were nevertheless able to contribute to the information about the range of parameter values to be expected.

6.3 Time Step Choice

The time interval at which a catchment model performs its calculations should be as large as possible to minimise the effort of computation. On the other hand, the larger the time interval the less faithfully are the continuous rainfall and riverflow functions approximated by average rates or volumes. Further, the method of solution employed may impose its own limits on the time step to maintain accuracy or stability.

A criterion for adequate representation of the rainfall

Table 6-2: Summary of Storm Period Data

Storm Group	Storm Code	Start Day	Length Days	Rainfall Inches	Peak Rain In/Hr	Peak Flow In/Hr
MAKARA-10					Hourly Mean Values	
1	K	12/6/64	3	2.67	0.28	.033
	AA	12/2/65	4	1.12	0.47	.008
	AB	1/3/65	9	4.08	0.51	.062
	BA	22/6/65	12	4.76	0.34	.097
	BB	18/8/65	10	4.21	0.28	.074
	CA	31/10/65	10	5.61	0.39	.149
2	H	13/10/60	2	1.58	0.38	.009
	I	18/9/61	4	1.70	0.20	.037
	DA	6/1/63	6	2.85	0.42	.011
	EA	2/6/63	9	4.24	0.45	.052
	FA	15/7/63	5	2.60	0.38	.036
	FB	5/8/63	7	5.20	0.42	.170
REYNOLDS					$\frac{1}{4}$-Hourly Mean Values	
1	HA	8/4/68	6	16.48	2.35	.290
	HB	22/4/68	7	2.49	0.36	.032
	EE	9/9/70	5	2.71	0.47	.078
	FF	14/10/70	5	2.50	0.64	.048
	G	29/5/71	6	4.45	0.53	.044
2	HC	6/7/68	16	5.51	0.36	.083
	B	8/5/70	8	3.78	0.42	.040
	CA	29/6/70	10	4.70	0.48	.088
	CB	12/7/70	20	6.44	1.44	.031
MOUTERE					Hourly Mean Values	
1	A	31/7/67	17	7.47	0.41	.174
	BB	16/11/67	5	5.90	0.36	.068
	CC	4/6/68	5	3.26	0.74	.160
	D	21/7/68	8	4.10	0.27	.034
	F	16/12/69	6	3.58	0.30	.020

and riverflow is proposed in this section, and the time intervals required to achieve this are determined by a consideration of the averaging process involved. These time intervals are also confirmed by examining the peak values of the Unitgraphs for various durations. Lastly the operation of the AM is checked to ensure it can operate satisfactorily at the chosen time intervals.

A. Averaging Error

The input and output of a catchment are both continuously-varying functions of time. Since a model operates at discrete time intervals the input and output are represented by volumes (or rates) corresponding to successive intervals (or instants) of time; in the case of volumes this amounts to taking the average rate over the time interval. The error involved in this approximation, illustrated in Figure 6-5, may be reduced to an acceptable level by decreasing the time interval. It now remains to define "an acceptable level" of error.

Since the task of the model is to simulate riverflows, the time interval choice is based on an acceptable level of error in the riverflow representation. It is assumed that the rainfalls will be sufficiently well represented at this same time interval. This assumption is encouraged by the observation that the higher-frequency rainfall fluctuations which may be masked by the averaging process do not produce discernible effects in the riverflow.

The acceptable level of error adopted required that the averaging process should reduce the peak rate by less than 2% on 80% of the peaks of the recorded riverflows.

Assuming the flow recorders were able to detect the

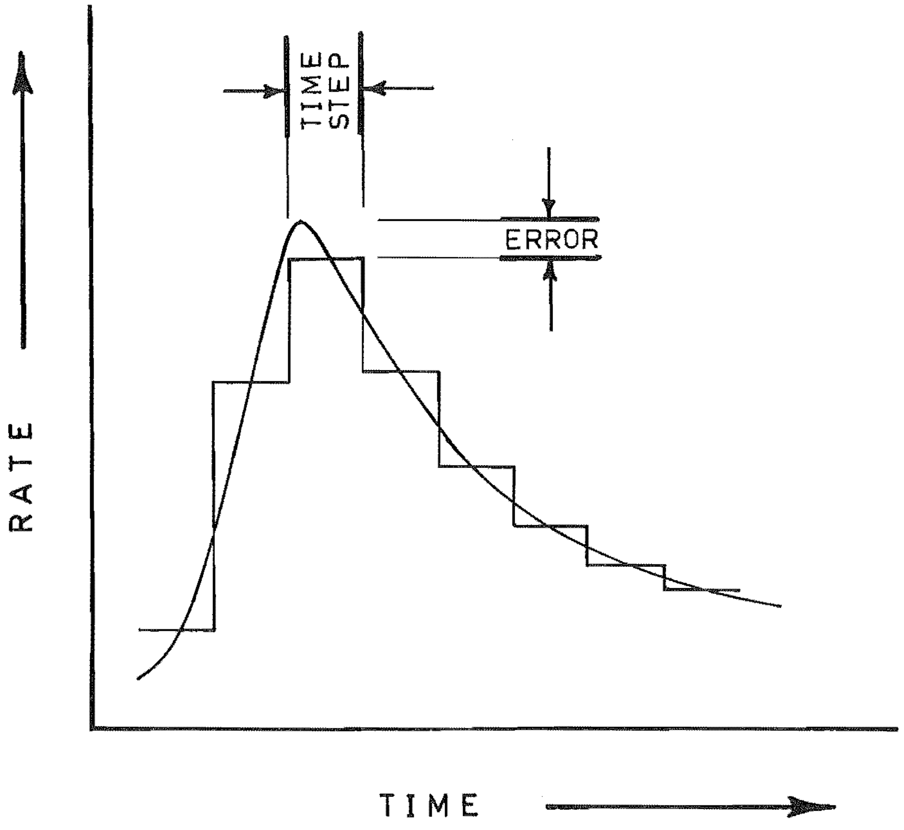


FIGURE 6-5: THE ERROR RESULTING FROM DISCRETISATION

true peaks, the errors in the peaks of the 26 riverflow records, resulting from the choice of various time intervals, were calculated. For each of the three catchments the time intervals corresponding to the above error criterion were found by graphical interpolation (see Table 6-3). These were used to select the actual intervals to be used for the model operation. These actual intervals, which were chosen for convenience to be submultiples of a day, are also shown in Table 6-3. The time interval called for by the error criterion for the Moutere catchment had to be slightly exceeded because the rainfall charts could not be read to the required accuracy.

Table 6-3: Choice of Time Step

CATCHMENT	TIME INTERVAL BY ERROR CRITERION	TIME INTERVAL ADOPTED
Makara-10	90min	1 hour
Reynolds	18min	$\frac{1}{4}$ hour
Moutere	40min	1 hour

B. Unitgraph Peaks

The reduction in the Unitgraph peak flow as the rainfall excess duration is increased also serves as a measure of the error incurred by representing rainfall and riverflow as volumes in successive discrete time intervals.

The response to a rainfall excess of one inch falling in one hour is the One-hour Unitgraph with a peak flow of q_1, say. By linear theory the response to the same volume of rainfall excess spread over two hours is the Two-hour

Unitgraph, which has a lower peak flow of q_2 (see Figure 6-6). The difference between q_1 and q_2 is a measure of the error incurred by representing the one-inch, one-hour rainfall excess as a volume averaged over a two-hour period. This is an upper limit of the error to be expected in real situations since it is assumed that no rain at all fell in the second hour.

Unitgraphs for suitable storms on each catchment were constructed for durations of fifteen minutes and one hour. It was expected that if a time interval of one hour was satisfactory for representing the riverflow peak, the average One-hour Unitgraph peak would not be more than 10% lower than the average Quarter-hour Unitgraph peak, taken as an approximation to the peak of the "instantaneous" Unitgraph. The larger error of 10% (compared with the 2% level in Part A above) was justified by the upper bound nature of this time-interval analysis.

The average Quarter-hour Unitgraph peak values and the percentage reductions of the One-hour Unitgraph peak values are shown for each catchment in Table 6-4. The maximum reduction of 11% was only just above the acceptable limit proposed above so the time intervals already chosen, of one hour or less, were deemed to be satisfactory.

Table 6-4: Unitgraph Peak Reduction for the Three Catchments

CATCHMENT	QUARTER-HOUR UNITGRAPH PEAK	ONE-HOUR UNITGRAPH PEAK REDUCTION
Makara-10	0.16in/hr	11%
Reynolds	0.38in/hr	9%
Moutere	0.21in/hr	6%

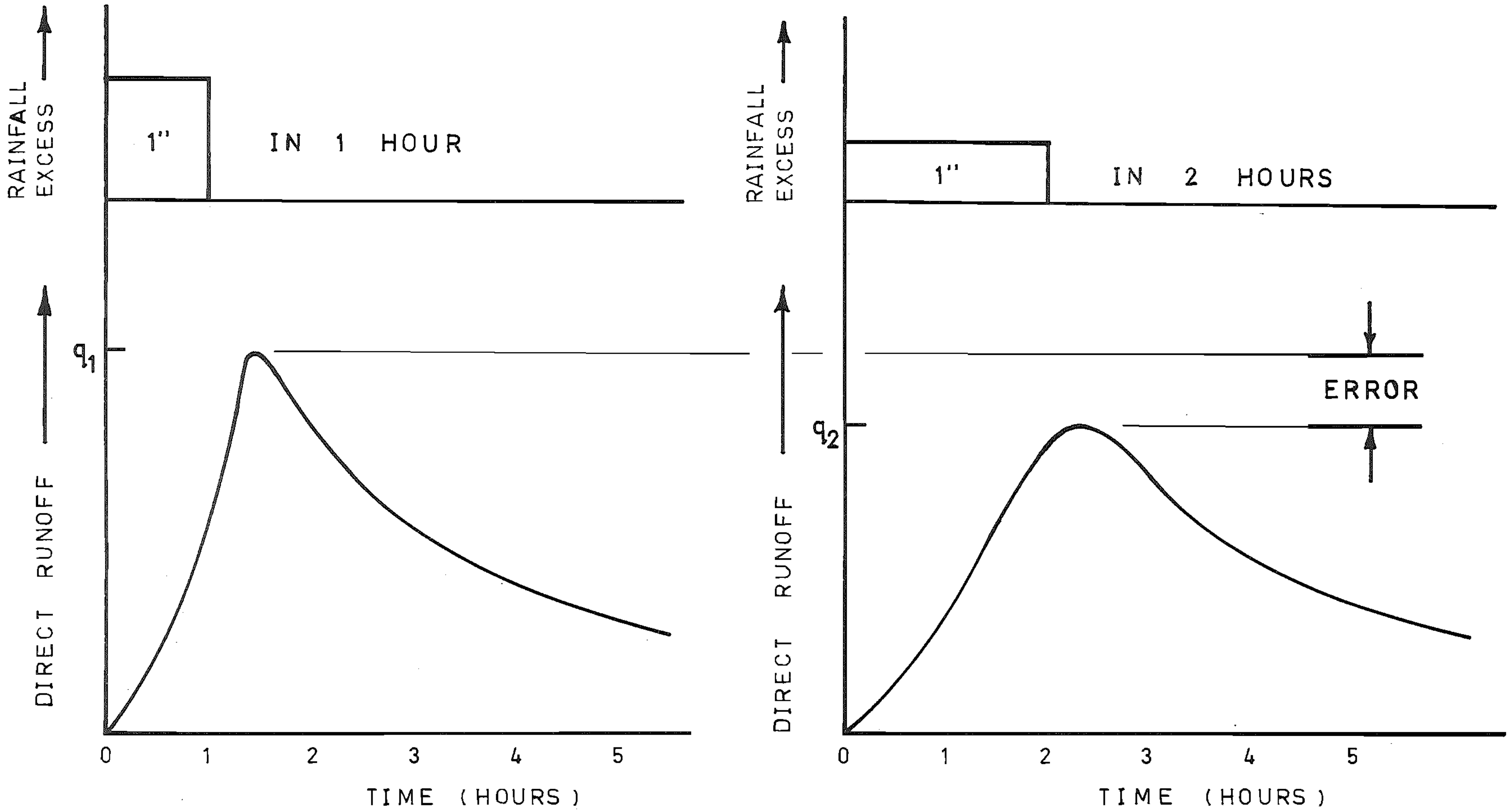


FIGURE 6-6: DECREASE IN UNITGRAPH PEAK WITH INCREASE IN EXCESS DURATION

Put another way, by linear theory the peak flow resulting from a quarter-hour burst of rainfall would be reduced by not more than 11% if the rainfall could only be resolved to an hour.

The values of the Unitgraph peaks confirm the relative responsiveness of the three catchments as indicated by the time interval criterion of Part A above. Reynolds exhibits the sharpest peaks, followed by Moutere and then Makara-10, as defined either by the Unitgraph peak or the required time interval.

C. Equation Solution

Having chosen the time intervals to suit the data it must be ensured that the model, and in particular the numerical solution to Richards' equation, will operate satisfactorily at these time intervals. Initially, numerical instability was observed in the numerical solution but changes in the finite-difference scheme (see Section 5.4B) cured this. Accuracy could be measured by a water balance between the input, output and change in storage within the model, and also by examining the change over a time interval of the coefficients C and K in the numerical solution (also discussed in Section 5.4B).

After the best values for the model parameters had been selected, it was found that the water balance error for all storm periods averaged 2.9% of the storm rainfall volume. Further, detailed printouts of the records with the greatest rainfall fluctuations revealed the differences between the values of C and K at the start of a time interval (as used in the solution) and the average values of C and K over the interval (which would ideally be used). The maximum difference in the values of C was 5.2%, and in the values of K, 3.2%.

Although these figures are satisfactory for a pilot study any further work should incorporate an iteration designed to seek average values of these coefficients over each time step. Smith and Woolhiser[26] describe this sort of iteration.

6.4 Parameter Estimation

A. Parameter Description

Not all the catchment properties which are incorporated in a catchment model can be estimated by inspecting the catchment; there are usually several which have to be evaluated by trial-and-error adjustment to fit the model output as closely as possible to the corresponding recorded flows. The AM has four such parameters, compared with five for the version of the SWM on which it is based. This section describes the AM parameters, and presents a sensitivity analysis of the model output with respect to changes in the parameters which cannot be estimated beforehand. Different methods of evaluating these parameter values are discussed leading to the choice of a manual adjustment technique based on the simultaneous plotting of both simulated and recorded riverflows.

The AM has a total of eleven parameters which are listed in Table 6-5. Although a primary modelling aim is to achieve sufficient realism so that the parameters can be measured or estimated using knowledge of the catchment alone, the simplification of the one-dimensional assumption prevents this. However most of the parameter values can be estimated by the methods discussed below; the resulting values for the

Table 6-5: Summary of Amended Model Parameters

Model Component	Program Identifier	Description
Interception	X(7)	Average Interception Storage Capacity of the vegetation (inches of water)
Evaporation	X(8)	Maximum Daily Evaporation (inches per day)
	X(9)	Moisture Content of the land surface (as a fraction of the saturated moisture content) below which evaporation ceases (dimensionless)
Infiltration and Subsurface Flow	X(3)	Saturated Moisture Content Factor (dimensionless)
	X(6)	Saturated Hydraulic Conductivity Factor (hours per inch)
	X(10)	Soil Column Lower Boundary Outflow ProportionalityConstant(dimensionless)
	X(11)	Subsurface Flow Leakage Fraction (dimensionless)
	X(12)	Soil Column Depth Factor (inches)
Surface Flow	X(13)	Manning's "n" (dimensionless)
	X(14)	Maximum Surface Flow Distance (feet)
	X(15)	Average Surface Slope (dimensionless)

Table 6-6: Parameter Values for the Amended Model Estimated Before Fitting

Parameter	Makara-10	Reynolds	Moutere
X(7)	0.10in	0.10in	0.15in
X(8)	.5in/day	.5in/day	.5in/day
X(9)	0	0	0
X(3)	200	200	200
X(13)	0.3	0.3	0.3
X(14)	390ft	2500ft	2500ft
X(15)	0.58	0.40	0.10

three catchments appear in Table 6-6.

X(7) (Computer Program Identifier): Interception Capacity

Crawford and Linsley[17] give a guide for this parameter based on the catchment cover, or basic research into interception may be consulted. The former was used here.

X(8), X(9): Evaporation Parameters

These values were estimated from the work of Boughton[18], from whose model the evaporation component was adapted. Such "borrowing" of values was permissible only because evaporation was expected to play a small part in a peak model: subsequent observation of the effect of varying these parameters confirmed this.

X(3): Soil Moisture Content

The saturated moisture content for a soil may be found by weighing a sample in both the wet and dry states. The Department of Scientific and Industrial Research[44] has done this for 54 soils in New Zealand, finding values between 0.30 and 0.85 by volume. However none of the DSIR samples was taken from within or near the catchments under study here, nor could any of the soil descriptions be matched to those of the catchments. Hence the moisture content factor for all three catchments was chosen to approximate the median DSIR value of 0.60.

X(13): Manning's "n" for Surface Flow

Crawford and Linsley[17] give a guide for this parameter for various types of surface. Alternatively most texts on Open Channel Flow contain tables for estimating Manning's "n". The value here was chosen using the former reference.

X(14), X(15): Maximum Surface Flow Distance and Slope

The maximum surface flow distance was found by measuring on a map a number of flow paths, perpendicular to the contours, from a ridge to a perennial channel. The mean value was adopted. Similarly the average slope of these same flow paths was adopted for the surface slope.

This leaves the four parameters whose values cannot easily be estimated. These are listed below:

X(6): Saturated Conductivity Factor

No mention of the systematic measurement of the conductivity of New Zealand soils was discovered.

X(10): Soil Column Lower Boundary Outflow Proportionality Constant

This value could be calculated from equation ~~5-4~~ 5-19 if all the quantities on the right-hand-side were known. However, neither the ratio of horizontal to vertical conductivity (not necessarily equal to one) nor the effective depth of subsurface flow can be readily estimated.

X(11): Subsurface Flow Leakage Fraction

This parameter exists to balance flow volumes with those recorded over long periods; without detailed knowledge of the geology of the catchment it can only be estimated by comparing simulated flow with recorded flow.

X(12): Soil Column Depth Factor

This factor gives the soil column height when multiplied by the number of distance increments in the numerical equation solution. Since it is an effective value, designed to match the behaviour of the simplified one-dimensional column to that of the true three-dimensional prototype, its estimation beforehand is not possible.

These are the parameters which were evaluated by trial-and-error adjustment to improve the agreement between the model output and the corresponding recorded riverflows. The next part of this section describes the effect that each of these parameters has on the model output.

B. Sensitivity

Before attempting to calibrate the model against recorded riverflows, it was first tested under a variety of conditions on a single storm period. A wide range of combinations of the four unknown parameters was used to explore the behaviour of the model, which could be readily assessed via the plotting of both simulated and recorded riverflows on the same graph. A consistent initial moisture status was specified for each simulation by setting the initial water table to the lower end of the soil column.

It was quickly observed that the subsurface flow contribution to the simulated flow would have to be quite high if the simulations were to avoid having a very low rise and recession, with an extremely high peak between (see Figure 6-7). Subsurface flow could be increased by increasing the lower boundary outflow proportionality constant X(10), or by increasing the hydraulic conductivity via X(6); since X(10) is restricted by the arguments of Section 5.4B to be of the order of a few percent, the saturated conductivity had to be increased. This in turn resulted in conductivities higher than the rainfall intensities of any of the 26 storm periods. Saturation of the surface, and therefore surface flow, could consequently only occur by exhaustion of the storage within the soil column, accompanied by a rising water table.

Next the effect of changing each of the four unknown parameters was explored in more depth by performing a graphical sensitivity analysis on one storm period on each catchment. Large changes were made to the parameter values

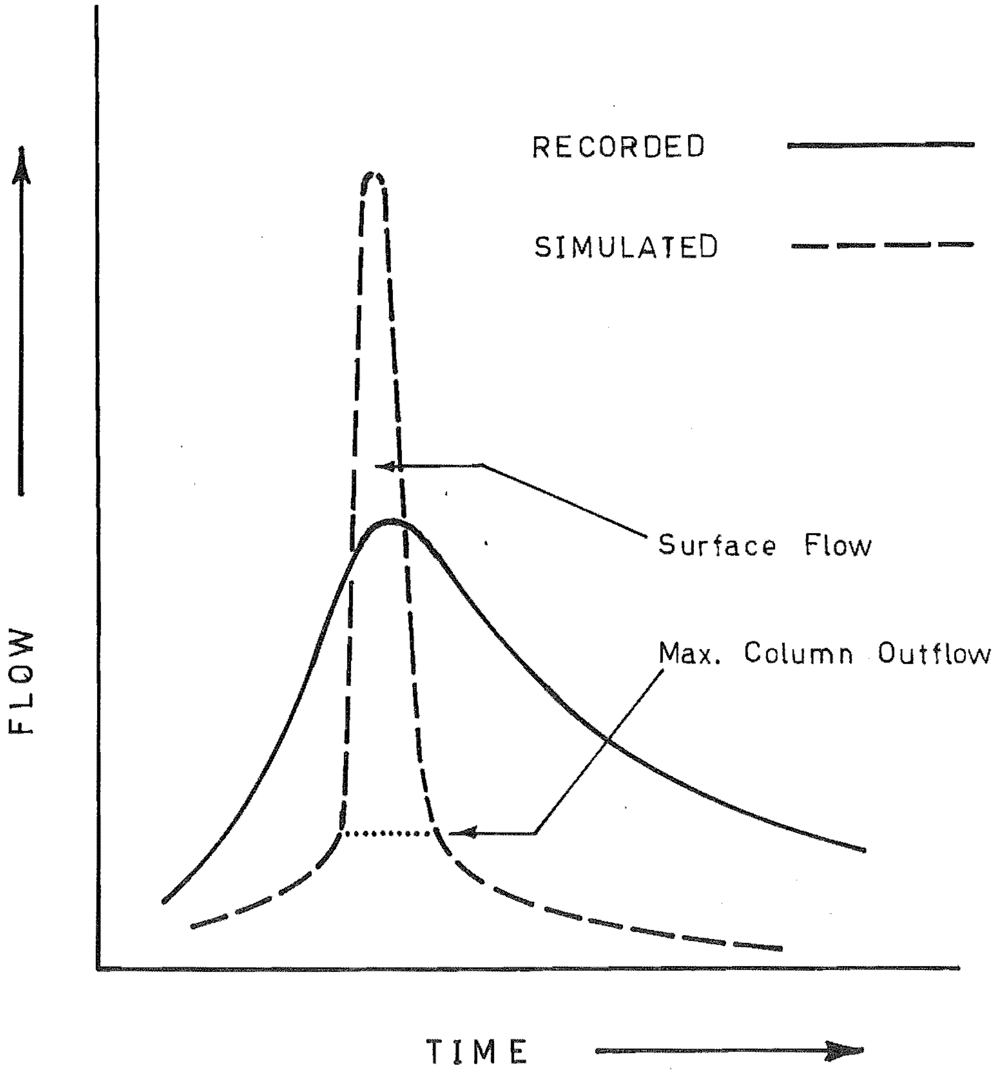


FIGURE 6-7: EFFECT OF INSUFFICIENT SUBSURFACE FLOW ON THE AMENDED MODEL SIMULATION

(to the extent of doubling or halving) and the effect of the change was displayed by plotting the "new" simulated hydrograph on top of the "old" simulated hydrograph. The sequence of operations is shown in Figure 6-8, and the result for the Moutere catchment appears in Figure 6-9 and Table 6-7. The effect of each parameter is now discussed in detail.

Table 6-7: Sensitivity Analysis for the Amended Model

Parameter	From	To	Effect on Simulated Hydrograph	On Volume
X(6)	30	60	(Conductivity from 5.3 to 2.6 in/hr) Slower rise & recession, but surface flow caused which boosts peak by 44%. Surface flow increased from 0 to 4% by volume	-11%
		15	(Conductivity from 5.3 to 10.5 in/hr) Faster rise, higher peak (by 14%), similar recession	+25%
X(10)	5%	10%	Same as doubling conductivity (see above)	+25%
		2.5%	Same as halving conductivity (see above)	-11%
X(11)	0.5	1.0	Eliminates flow entirely, since no surface flow with "old" parameter set	-100%
		0.0	Doubles all flow	+100%
X(12)	1.0	2.0	Slower rise, lower peak (by 62%) and slower recession	-32%
		0.5	Faster rise, higher peak (by 80%), faster recession. Surface flow 0.8% by volume instead of zero	+10%
Catchment: Moutere			Storm: BB	

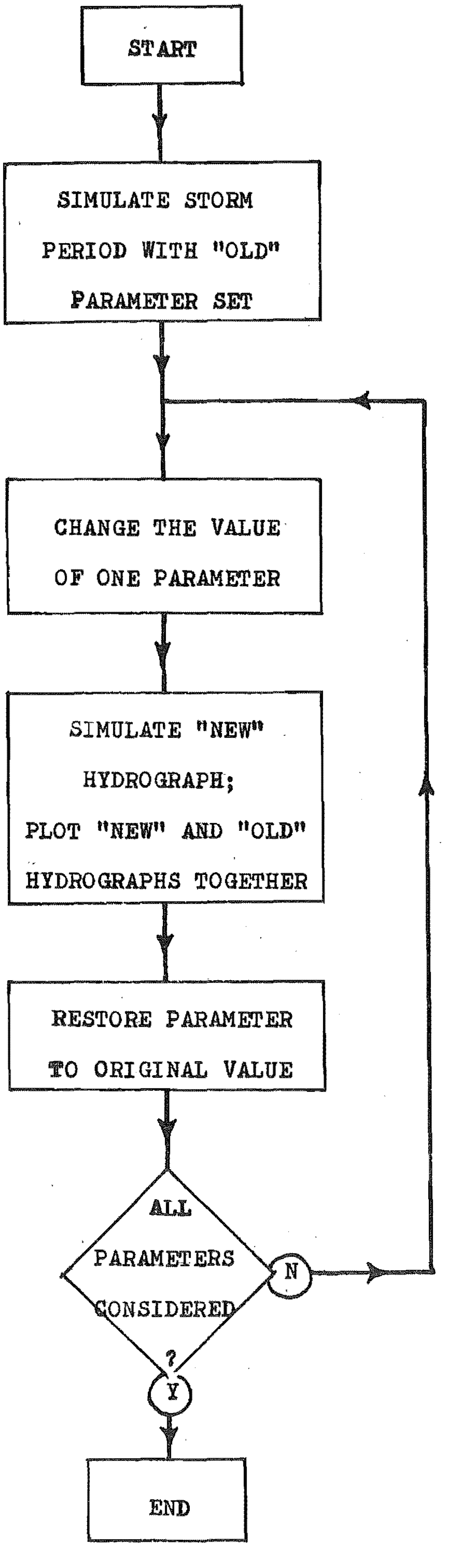


FIGURE 6-8: SENSITIVITY ANALYSIS SEQUENCE

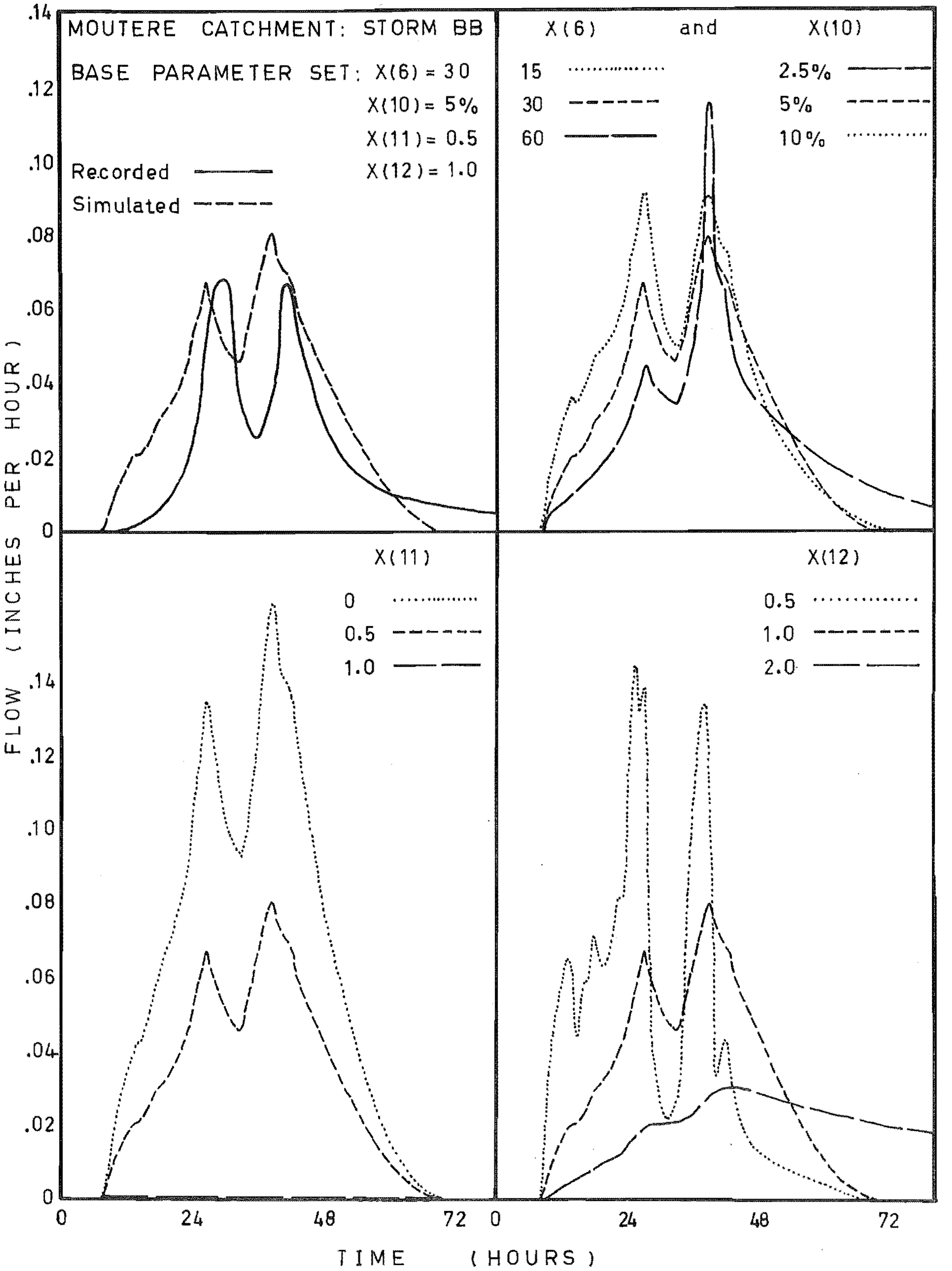


FIGURE 6-9: SENSITIVITY ANALYSIS FOR THE AMENDED MODEL

X(6): Saturated Conductivity Factor

The effect of changing this parameter depends on whether the parameter values and the rainfall intensities are such as to exhaust the storage in the soil column and allow the more rapid surface flow component to act.

(a) No Surface Flow Occurs. In this case halving the conductivity produces a slower rise, a lower peak, a slower recession and a decreased volume of simulated riverflow. This is because infiltrating water takes longer to reach the water table where it can contribute to subsurface flow, and because low conductivity means low soil column outflow for a given water table position, by equation 5-16. This low outflow is responsible for the persistence of flow after the storm.

(b) Surface Flow Occurs. In this case halving the conductivity produces a slower initial rise as before. But when the soil column becomes saturated the surface flow component causes a rapid rise to a higher peak, followed by an equally rapid recession. When surface flow has ceased the recession slows down, as in (a).

In both cases doubling the conductivity produces an effect of a similar magnitude in the opposite direction.

X(10): Soil Column Outflow Proportionality Constant

Whether or not the model simulates surface flow the effect of doubling or halving this parameter is almost identical to that of doubling or halving the saturated conductivity. This is because both forms of response simulated by the model, surface or subsurface, occur through a rising water table, saturation downwards from the surface having been prevented by the necessity to employ high values of conductivity. The water table rise occurs as a result of exhaustion of storage in the soil, controlled by the lower boundary velocity via equation 5-16 ~~5-4~~, on which the values of the proportionality constant and the conductivity have identical effects.

X(11): <u>Subsurface Flow Leakage Fraction</u>

Increasing this parameter causes a decrease in the simulated flow over the whole hydrograph, and vice versa. The magnitude of this decrease depends on the proportions of the simulated flow contributed by the subsurface and the surface components, ranging from no decrease (when the response comprises only surface flow) to a 100% decrease (totally subsurface flow and the leakage fraction is increased from 0 to 1).

X(12): <u>Soil Column Depth Factor</u>

Doubling this factor (that is, doubling the height of the soil column) has a distinct damping effect on the hydrograph. The rise is slower, the peak lower, and the recession slower. This is consistent with the greater depth water must travel to reach the water table, and the increased volume of water required to saturate the column and initiate surface flow. Halving the depth factor has the opposite effect, achieved both by increasing the response speed of the soil column and also, by reducing the storage to be satisfied before saturation, encouraging surface flow.

In the absence of surface flow the effect of halving the soil depth is similar to that of doubling the conductivity. But whereas halving the soil depth <u>encourages</u> surface flow, doubling the conductivity <u>discourages</u> it. This provides a fundamental difference between these two parameters which could be valuable during parameter evaluation.

Unfortunately this sensitivity information is particular to the current values of all ten other parameters. (The dual effect of the conductivity in the presence or absence of surface flow illustrates this.) So an examination of the interaction between the parameters which could not be estimated in advance was made to supplement the sensitivity study. Fortunately all four of these parameters did not need to be considered. The similar behaviour of the conductivity and the proportionality constant meant that only one of these needed to be included; the proportionality constant was chosen for this. And in view of the readily-understood action of the subsurface leakage fraction this parameter was omitted. Therefore the interaction test considered a number of combinations of the values of the proportionality constant and the soil column depth factor. As for the sensitivity study, the seven parameters able to be estimated beforehand were given the values shown in Table 6-6.

To aid in the interpretation of the results of this test two numerical indices of performance were calculated for each simulation. This enabled the interaction to be displayed via contours of equal performance on a graph with the two parameter values on the axes; the optimum was then the lowest, or highest, point of the surface defined by these contours. The two indices used were the percentage error in the peak simulated flow (for which the optimum value is zero), and the coefficient of variation proportional to the square root of the sum of the squares of the differences between the simulated and recorded flow ordinates, taken each time step of the simulation. The sum of the squares of the differences is commonly used in hydrology to quantify the goodness of fit between two functions of time; the larger the value the worse the fit, with zero being a perfect fit.

A range of combinations of the proportionality constant and the depth factor, spanning a ratio of 128 in each parameter, was explored using several storms on the Moutere catchment. The region of the optimum coefficient of variation is shown for a typical storm in Figure 6-10; outside this region the coefficient steadily worsened in all directions. The storm is the same one shown in Figure 6-9, and the pattern was similar for the other storms tested.

According to the coefficient of variation an optimum exists at a value of 1.1 inches for the depth factor, and 3.5% for the proportionality constant. Moreover, the line of zero peak error for the second of the two peaks in the storm passes close to this point. It is not possible to correctly simulate both peaks simultaneously anywhere in the region examined.

The dual nature of the conductivity noted in the sensitivity study was confirmed here. Referring again to Figure 6-10, in the no-surface-flow zone contours of the coefficient of variation slope up to the right. This also occurred with the contours of peak error. In other words, the two parameters have <u>opposite</u> effects, since an increase in one can be negated by an increase in the other. On the other hand, when the model predicts surface flow the contours run generally up to the left. In this zone the two parameters have <u>similar</u> effects, since an increase in one can be balanced by a decrease in the other, to maintain a similar performance.

Having thoroughly examined the model behaviour in a general manner, it was then possible to attempt to find the

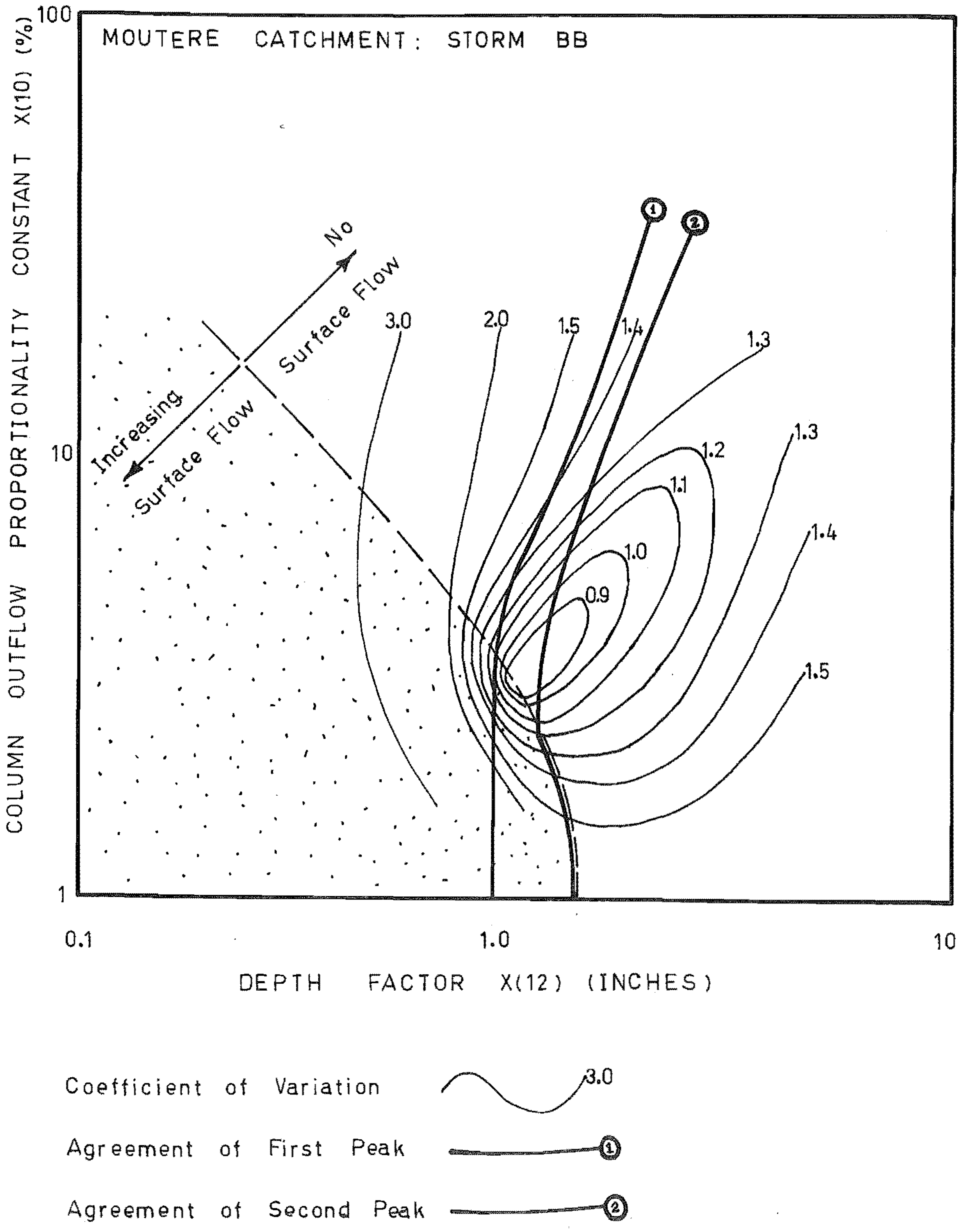


FIGURE 6-10: INTERACTION BETWEEN THE PROPORTIONALITY CONSTANT AND DEPTH FACTOR FOR THE AMENDED MODEL

best combination of parameters for a group of storms. Because of the imperfections in this and any model, the best parameters on the whole would not necessarily coincide with the best parameters for a particular storm.

C. Evaluation Methods

All parameter evaluation methods which attempt to duplicate the recorded riverflows are "trial-and-error" methods; differences occur only in the way in which past experience is used to choose new parameter values for trial. This section examines various ways which may be used to choose the new parameter values.

The most basic method uses the plotting of the simulated and recorded riverflows on the same graph. Differences between the two can be observed directly and sensitivity or interaction information used to determine which parameter changes are required for improvement. Features of this method are:

(a) "Goodness of Fit" evaluated by inspection of the two graphs by the user automatically reflects the purposes for which the model is to be used, for example an emphasis on the agreement between the peak flows. On the other hand this subjectivity will allow different persons to obtain different "best" parameter sets.

(b) Where several parameters seem to affect the simulation in a similar way, physical reasoning (which fundamentally underlies conceptual modelling) can be used to decide which parameters should be changed to effect a certain improvement. Again this subjectivity can also be seen as a disadvantage.

(c) Because of human intervention the method requires less computer time, but more total time, to determine the best fit.

(d) It may be difficult to decide which is the better of two different simulations, so that making use of past experience is not straightforward.

(e) It is difficult to use on a group of storms when the sensitivity information may indicate conflicting parameter changes to improve different storm simulations.

Some of these difficulties may be reduced if the success of a simulation can be described by a number such as an index of fit between the recorded and simulated hydrographs, even though no single number can adequately incorporate all the features of the disagreement between two graphs. Some possible indices of the closeness of fit between two regularly-tabulated functions of time are described in Table 6-8, together with the particular feature of the fit which each one emphasises. Each of these indices was available from the computer printout of a simulation by the AM.

Given a suitable index of fit the trial-and-error process involves changing parameter values, either manually or automatically, in order to improve the index. Automatic methods for doing this usually examine the partial derivatives of the index with respect to each of the parameter values in order to determine the direction, or combination of parameter changes, in which most improvement may be expected. For example the "Steepest-descent" minimisation method moves in the direction of steepest slope by changing parameter values in proportion to the corresponding partial derivatives. The differences between different automatic optimising schemes arise from different

Table 6-8: Indices of Fit Between Two Functions

Name	Evaluation	Emphasis	Good Fit	Bad Fit
Peak Error	$\frac{X_{max} - Y_{max}}{Y_{max}} \times 100\%$	Major Peak Only	0	$\pm\infty$
Volume Error	$\sum_{i=1}^{n}(X_i - Y_i)$	Volume Only	0	$\pm\infty$
Sum of the Squares	$\sum_{i=1}^{n}(X_i - Y_i)^2$	Peaks(mildly), Volume(mildly)	0	∞
Sum of the Fourths	$\sum_{i=1}^{n}(X_i - Y_i)^4$	Peaks(strongly), Volume(weakly)	0	∞
Coefficient of Correlation	$\frac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i-\bar{X})^2\sum_{i=1}^{n}(Y_i-\bar{Y})^2}}$	Linear Correlation	1	0
Coefficient of Variation	$\sqrt{\frac{\sum_{i=1}^{n}(X_i-Y_i)^2}{\bar{Y}^2 n}}$	Dimensionless Sum of the Squares	0	∞
Coefficient of Determination	$\sqrt{\frac{\sum_{i=1}^{n}(Y_i-\bar{Y})^2-\sum_{i=1}^{n}(X_i-Y_i)^2}{\sum_{i=1}^{n}(Y_i-\bar{Y})^2}}$	Proportion of VarianceExplain-ed by Simulation	1	$-\infty$

Where the x_i are n equally-spaced tabulated values of the function x, and the y_i of the function y.

x_{max}, y_{max}, $\bar{x}$ and $\bar{y}$ are the maximum and mean values of the x_i and y_i respectively.

ways of obtaining the maximum improvement in the index with the minimum number of evaluations of the index, each of which requires a complete simulation.

The numerical index approach has been used on several catchment models, including a simplified SWM, by Dawdy and O'Donnell[20], and on the Boughton model by Boughton[45]. Ibbitt and O'Donnell[46] have compared the performances of several automatic optimising methods using synthetic data and have suggested ways[47] in which model design can overcome various problems met when using real data. A steepest-descent optimiser was also used by this writer[31] in the course of finding the optimum parameter set for daily simulation by the SWM for five catchments in New Zealand. Based on this experience the numerical index technique can now be assessed for possible use in this study:

(a) Once an index of fit has been chosen the trial-and-error process becomes completely objective. But if the index does not embody the desired features this objectivity is illusory. For example if the prediction of peak flow rate was more important than the time of the peak, the Sum of the Squares would not be suitable because it depends unduly on the coincidence in time of the peaks. This can be illustrated by calculations on pairs of symmetrical, triangular hydrographs (Figure 6-11): a time shift of only 16% of the base width scores about the same as a hydrograph everywhere 50% low or high.

(b) Use of automatic optimisation techniques makes the trial-and-error process rapid in total time. But this is gained at the expense of high computer time use,

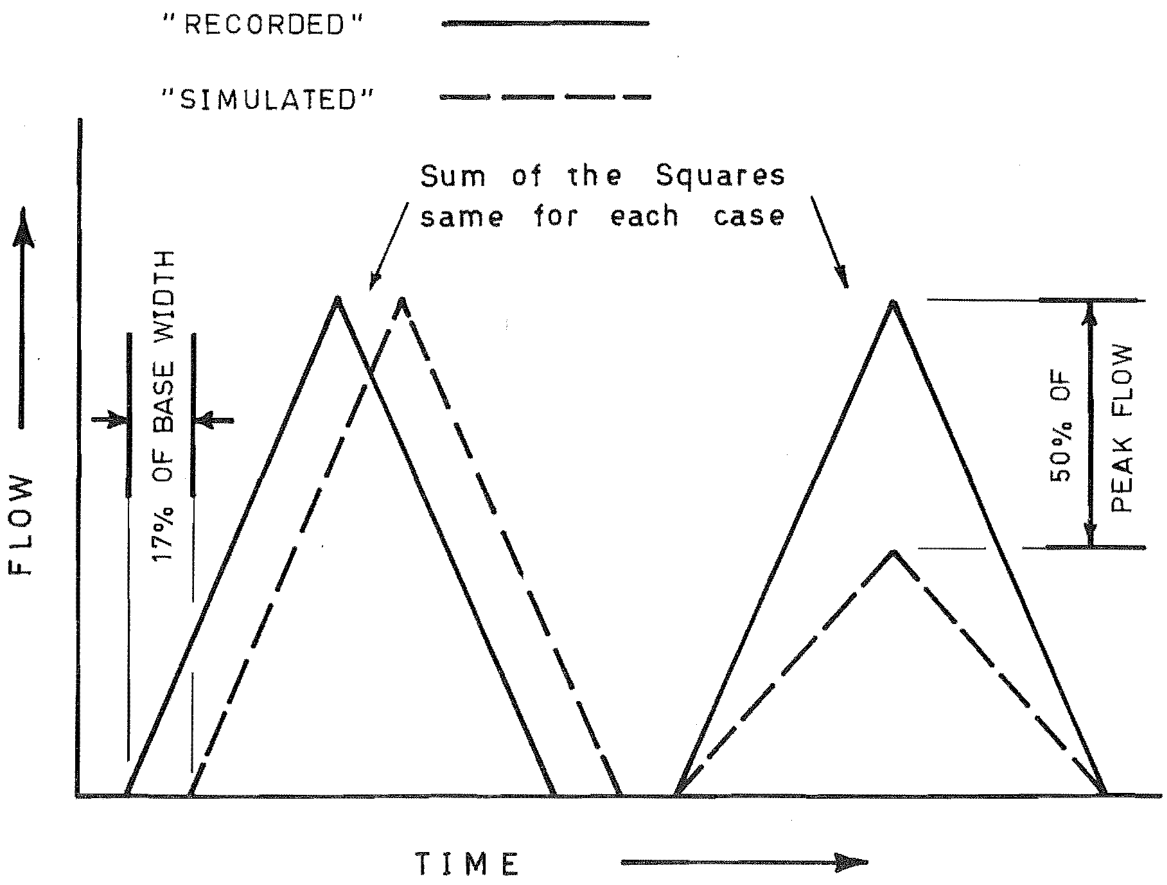


FIGURE 6-11: DEPENDENCE OF THE SUM OF THE SQUARES ON THE COINCIDENCE OF PEAKS

since many unproductive simulations must be made to establish the partial derivatives.

(c) Compromise between the conflicting demands of several storm periods on the same catchment becomes possible, since an overall index can be defined as the sum, or some other combination, of several individual storm indices of fit.

(d) The surface representing the index in the multidimensional parameter space (see Figure 6-10 for a two-dimensional parameter space) is not well suited to optimum-seeking unless the model has been designed with this in mind. Imperfections of both model and data create many "valleys" and "potholes", which make the global optimum impossible to find with a numerical search routine. Neither the SWM nor the AM was designed to circumvent these difficulties.

D. Method Adopted

Having discussed the two major types of parameter evaluation method a choice for the testing of the AM may now be made. Because the computing time of the AM was considerable (about 15 seconds for a four-day hourly simulation), a method which minimised the number of simulations required was desirable. In addition close control over the parameters was felt to be important for a physically-based model. Therefore the manual parameter adjustment method, using simultaneous plotting of simulated and recorded riverflows, was adopted.

However some features of the numerical index method were incorporated, in that the coefficient of variation and the peak error (see Table 6-8) were recorded for each run.

Together these indices approximately agreed with estimates of success based on inspection of the graphs and enabled comparisons to be drawn between this and other models. Further, since the coefficient of variation is independent of the length of record and of mean flow rate (in the absence of serial correlation) an approximate scale of correspondence between the coefficient and a subjective judgement of performance is possible. With the comments of a colleague[48] Table 6-9 was drawn up. This enabled an absolute measure of the success of a simulation to be tentatively quoted.

Table 6-9: Correspondence between the Coefficient of Variation and a Subjective Assessment of Goodness of Fit

Subjective Assessment	Coefficient of Variation
Excellent	0 - .25
Very Good	.25 - .50
Good	.50 - 1.0
Fair	1.0 - 2.0
Poor	2.0 +

6.5 <u>Performance Evaluation</u>

A. <u>Comparison with Stanford Watershed Model</u>

In this section the AM parameters are evaluated by the method explained in the last section, using about half of the available records, as a test of the fitting ability of the model. Then the predicting ability was found by applying the model with the same parameters to the rest of the records. The results are compared with those of the same test carried out on the SWM, and the performance and parameter values obtained for each are discussed.

The first step in the comparative test was to evaluate the values of the four parameters of the AM which could not be estimated by inspection of the catchment; referring to Table 6-5 these were X(6), X(10), X(11) and X(12). This was done by the method described in Section 6.4D using five or six records on each catchment, being those described as "Group 1" in Table 6-2. The success of this step was a measure of the versatility of the model in <u>fitting</u> given records. The second step was to simulate the remaining records, those described as "Group 2" in Table 6-2, using the values of the parameters found above. This test parallels the situation in practice when the recorded riverflows would not be known. The success of this step was a measure of the <u>prediction</u> ability of the model. Although this "split-record" test is required to evaluate predicting ability the best possible estimates of the parameter values could only be obtained by using all the available records.

The success of both fitting and predicting was then compared with that of the SWM operating on the same records under the same conditions as the AM. In this way the effect of errors in the data, short record length and choice of initial conditions would be minimised. The version of the SWM used was the Model IV as described by Crawford and Linsley[17] with the snowmelt and channel flow components omitted, and was the same as that used by this writer in an earlier study for daily simulation[31]. This version has fourteen parameters of which nine may be estimated without recourse to recorded flows; the values adopted for these nine are shown in Table 6-10. The remaining five were

Table 6-10: Parameter Values for the Stanford Watershed Model Estimated Before Fitting

Parameter*	Description	Makara-10	Reynolds	Moutere
a_1	Impervious Area Fraction	0	0	0
a_2	Average Surface Slope	0.58	0.40	0.10
a_3	Maximum Surface Flow Distance	390ft	2500ft	2500ft
a_4	Manning's "n" for Surface Flow	0.3	0.3	0.3
x_1	Interception Capacity	0.10in	0.10in	0.15in
x_4	Lower Soil Storage Capacity	9.0in	9.0in	9.0in
x_5	Upper Soil Storage Capacity	0.6in	0.6in	0.6in
x_8	Lower Soil Evaporation Rate Factor	.23in/hr	.23in/hr	.23in/hr
x_{10}	Groundwater Recession Variation Factor	0	0	0

* These parameter symbols refer to Figure 3-4

evaluated by the method of Section 6.4D as for the AM, using the Group 1 storms.

To avoid introducing additional variables into the fitting process both models were made "dry" at the start of each simulation. In the case of the AM the water table was set at the bottom of the soil column and the interception storage was made empty. Similarly in the case of the SWM all storages representing the flow or retention of water were made empty.

B. Results

The values of the parameters found by the fitting process on the Group 1 records are shown in Table 6-11 (SWM) and in Table 6-12 (AM). An example of the simulation of a storm fitted by both models is shown in Figures 6-12 and 6-13; others appear later to illustrate particular points in the discussion. An example of a predicted storm is shown in Figures 6-14 and 6-15.

Table 6-11: Optimum Values of the Fitted Parameters for the Stanford Watershed Model

Parameter*	Description	Makara-10	Reynolds	Moutere
x_2	Infiltration Factor	.25in/hr	.50in/hr	.33in/hr
x_3	Interflow Ratio	4.0	3.0	3.0
x_6	Daily Interflow Recession Constant	.01	.01	0.1
x_7	Daily Groundwater Recession Constant	0.8	0.8	0.8
x_9	Groundwater Leakage Fraction	0.5	0.5	0.5

* These parameter symbols refer to Figure 3-4.

Table 6-12: Optimum Values of the Fitted Parameters for the Amended Model

Parameter	Description	Makara-10	Reynolds	Moutere
X(6)	Saturated Hydraulic Conductivity Factor	30hr/in	30hr/in	30hr/in
X(10)	Soil Column Outflow Proportion. Constant	10%	10%	10%
X(11)	Subsurface Flow Leakage Fraction	0.5	0.6	0.5
X(12)	Soil Column Depth Factor	0.8in	1.0in	1.0in

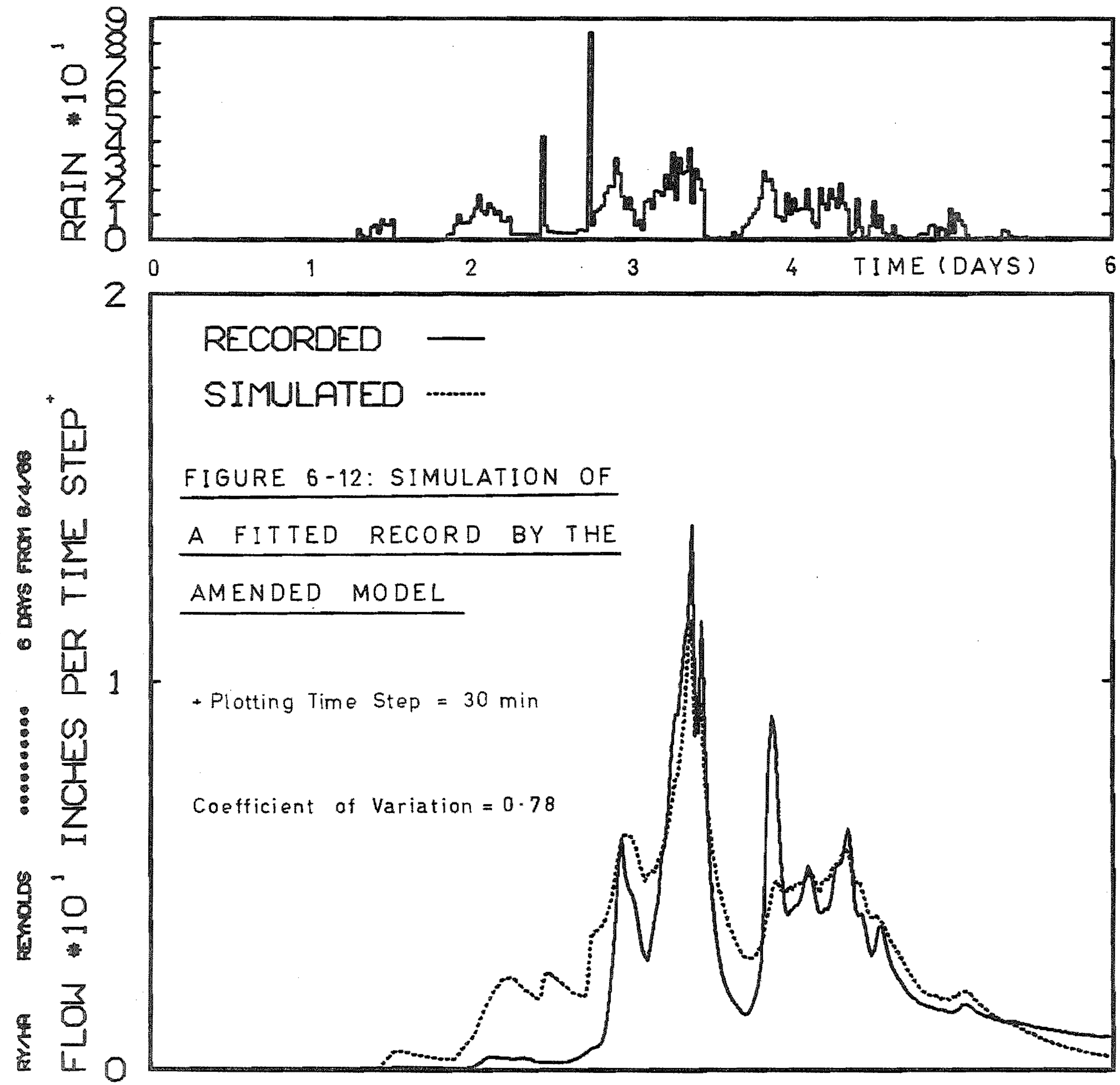


FIGURE 6-12: SIMULATION OF A FITTED RECORD BY THE AMENDED MODEL

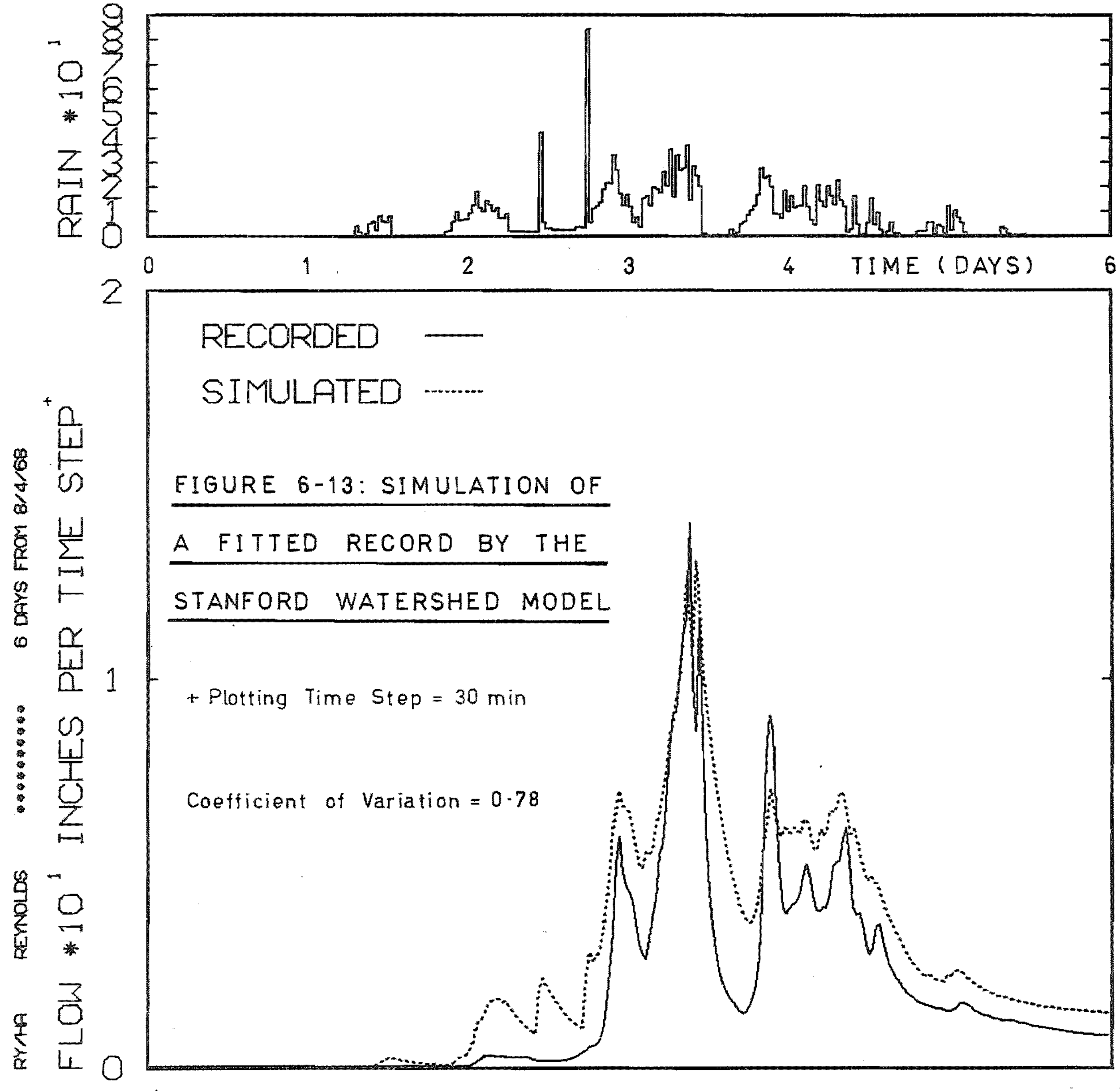


FIGURE 6-13: SIMULATION OF A FITTED RECORD BY THE STANFORD WATERSHED MODEL

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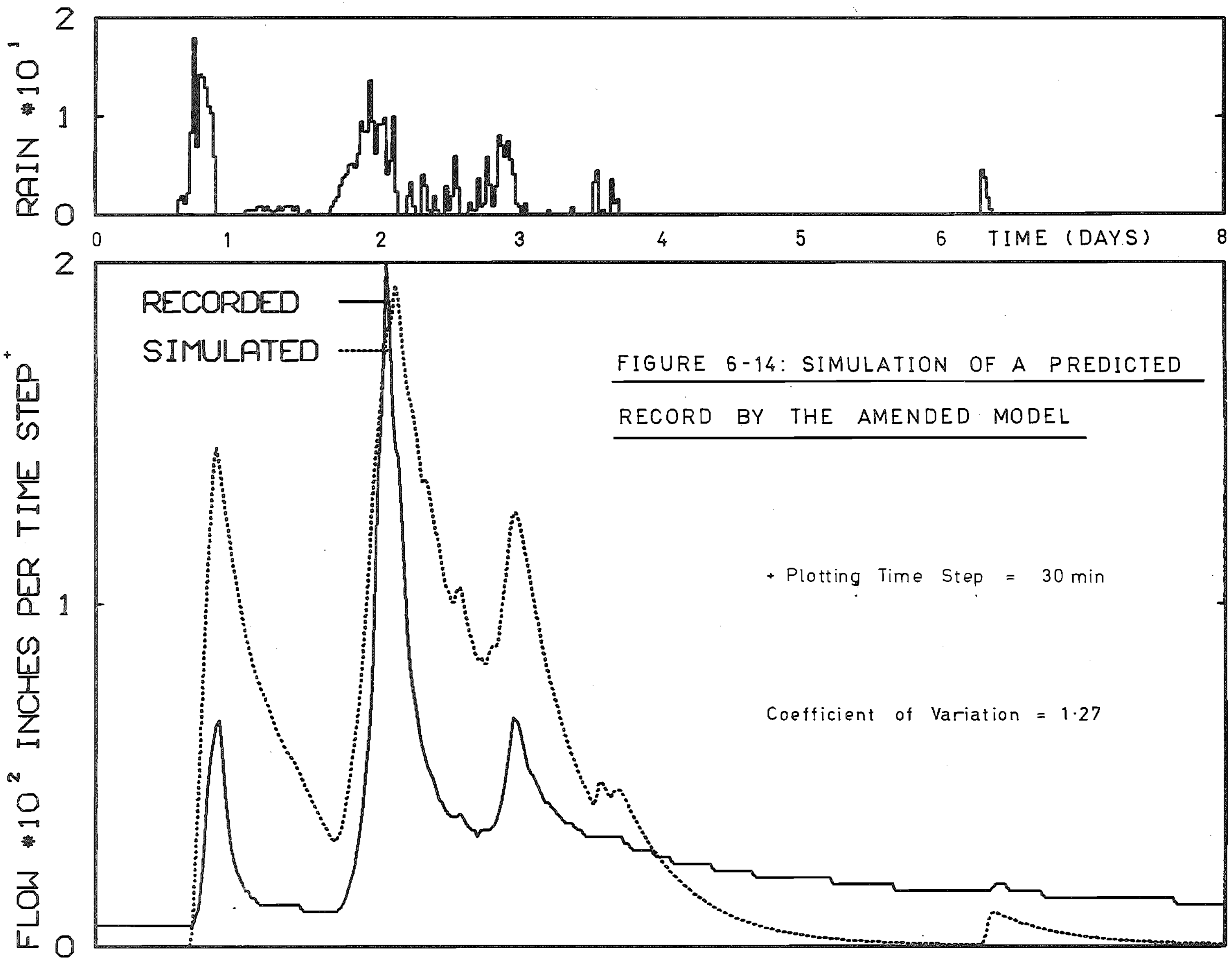

FIGURE 6-14: SIMULATION OF A PREDICTED RECORD BY THE AMENDED MODEL

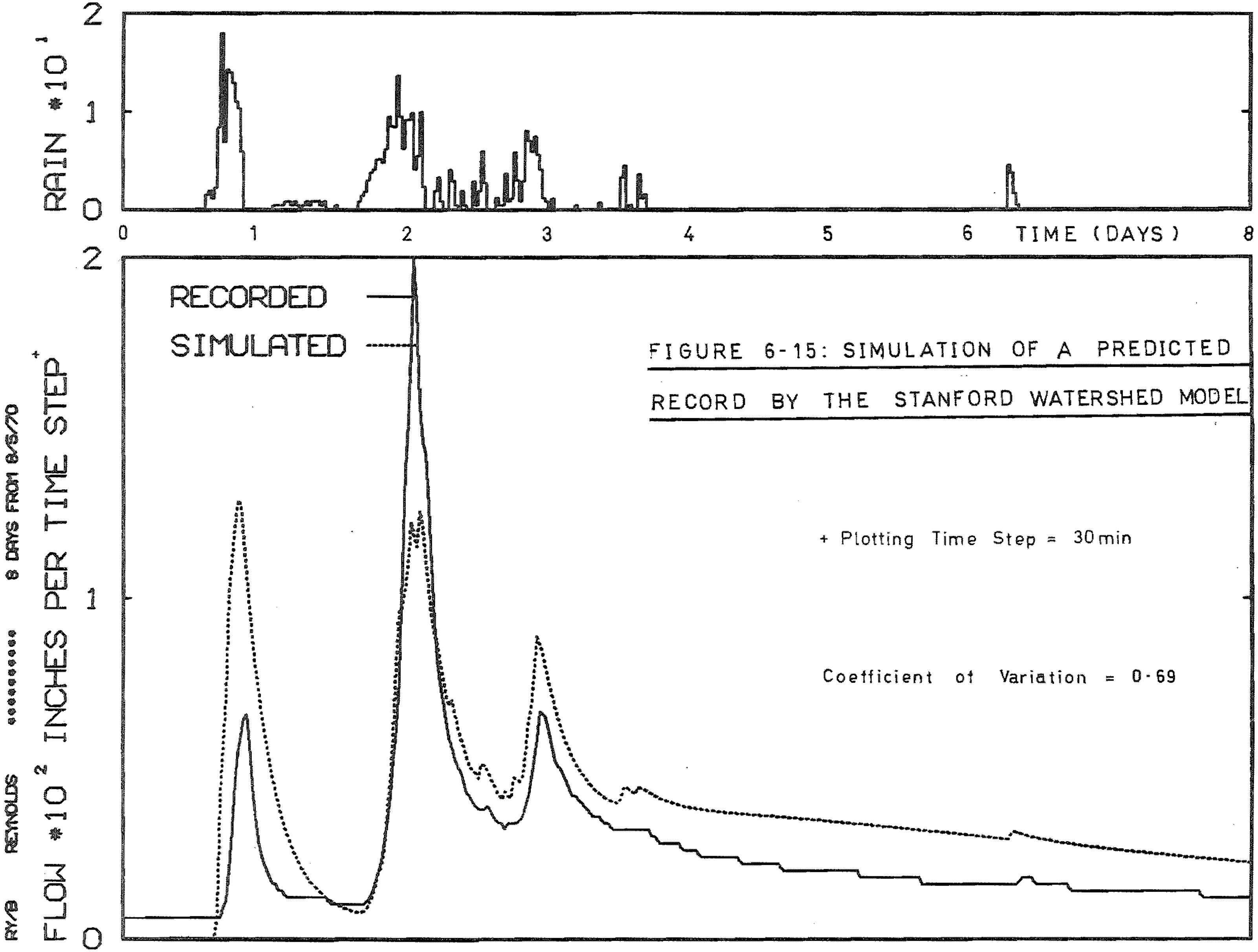

FIGURE 6-15: SIMULATION OF A PREDICTED RECORD BY THE STANFORD WATERSHED MODEL

Inspection of the graphs of simulated and recorded flows showed the two models to be remarkably similar, there being more difference between simulations by the same model than between models on the same storm. The major difference observed in the simulated hydrographs was that the AM recessions fell away too soon, and consequently the SWM recessions were better. Both models tended to predict high on early peaks in a record and low on the later ones. As was expected performance on the Group 2 records was generally inferior to that on the Group 1 records, but again there was little difference between the two models.

A comparison between the two models using the peak error and coefficient of variation defined in Table 6-8 is given in Table 6-13. The hazards of using numerical indices have already been discussed but comparisons are hard to draw without the use of numbers. This table confirms the result of the graphical comparison, there being little difference between the indices of fit for a given record. The coefficients of variation generally favour the SWM (because of its better recession simulation) but the peak errors indicate equal performance. Another assessment of the success of both models may be made using Table 6-9; this assessment, shown in Table 6-14, shows the very slight superiority of the SWM.

The parameter values found by fitting did not vary greatly between the three catchments for either model. It is not possible on this evidence to determine whether these values would be appropriate for a large proportion of New Zealand catchments, or whether these catchments just happen to be similar hydrologically. In the next section, after commenting on some aspects of the simulations, the

Table 6-13: Numerical Comparison of the Performance of the Stanford and Amended Models

Storm Group	Storm Code	Peak Error (% of Recorded Peak) Stanford	Amended	Coefficient of Variation Stanford	Amended
MAKARA-10					
1	K	+128	+160	1.98	2.87
	AA	+300	+300	6.64	7.23
	AB	+16	+6	1.30	1.41
	BA	-32	-21	1.02	0.81
	BB	-31	-12	0.89	0.57
	CA	-33	-29	1.30	1.31
	mean	90	88	2.19	2.37
2	H	+530	+720	5.16	7.28
	I	-41	+22	0.99	1.17
	DA	+290	+320	4.23	6.29
	EA	+59	+71	1.74	2.72
	FA	+78	+100	1.42	1.59
	FB	-24	-15	0.94	0.58
	mean	170	208	2.41	3.27
REYNOLDS					
1	EE	-14	-20	0.67	1.07
	FF	+6	-16	0.70	0.95
	G	+34	+8	1.71	2.32
	HA	-39	-19	0.78	0.78
	HB	-5	-16	0.72	0.61
	mean	20	16	0.92	1.14
2	B	-36	-4	0.69	1.27
	CA	-45	-37	0.46	0.77
	CB	+120	+68	0.74	0.81
	HC	-65	-57	0.69	0.72
	mean	66	41	0.65	0.89
MOUTERE					
1	A	-50	-34	1.25	1.09
	BB	+7	+33	0.89	1.45
	CC	-57	-34	1.37	1.10
	D	-17	+11	0.64	0.68
	F	+50	+180	1.01	2.82
	mean	36	59	1.03	1.43

Table 6-14: Subjective Comparison of the Performance of the Stanford and Amended Models

Subjective Assessment	Number of Storms	
	Stanford Model	Amended Model
Excellent	0	0
Very Good	0	0
Good	13	10
Fair	10	9
Poor	3	7
	26	26

significance of the parameter values found for each model is discussed.

C. Discussion

The object of replacing the subsurface components of the SWM by a solution to Richards' equation was to determine whether this would improve the performance. The use of the one-dimensional form of Richards' equation has been found not to improve the performance, but it has not reduced it either, in spite of the severe restrictions which the one-dimensional assumption implies. This result demonstrates that a fundamental description of the hydraulics of part of the hydrologic cycle, in spite of severe simplification in both formulation and in boundary conditions, can be incorporated into a model which will deliver essentially the same performance as that of a presently-accepted standard. At the same time the number of parameters requiring to be found by trial-and-error was able to be reduced. This indicates that more general formulations of Richards' equation could be very successful as model components if the required data could be measured.

More general formulations of this equation are discussed with this in view in the next chapter. Meanwhile the following specific aspects of the simulations are worthy of discussion:

(a) Early Peaks

Both models showed a tendency to over-simulate the early peaks in a record while the later peaks were too low (see Figures 6-16 and 6-17). This phenomenon had also been noticed when using the SWM on daily records[31] and the over-prediction was not able to be removed by varying any of the parameters except the interception capacity, the value for which was considered reliable. What is required is greater power to reduce the response of the model early in a storm without altering the later response.

The SWM contains two mechanisms for modelling this variable response. The first mechanism is an inverse relationship between the maximum infiltration rate and the volume of water in the lower soil storage of the model. As the storage fills during the course of a storm the maximum infiltration rate decreases. The second mechanism represents depression storage by preventing a fraction of the water on the surface from contributing to surface flow. The fraction decreases as the storm continues. The evidence presented here suggests that these mechanisms need to have a greater effect.

The AM contains only the former type of mechanism, in which the infiltration rate is reduced in the classical

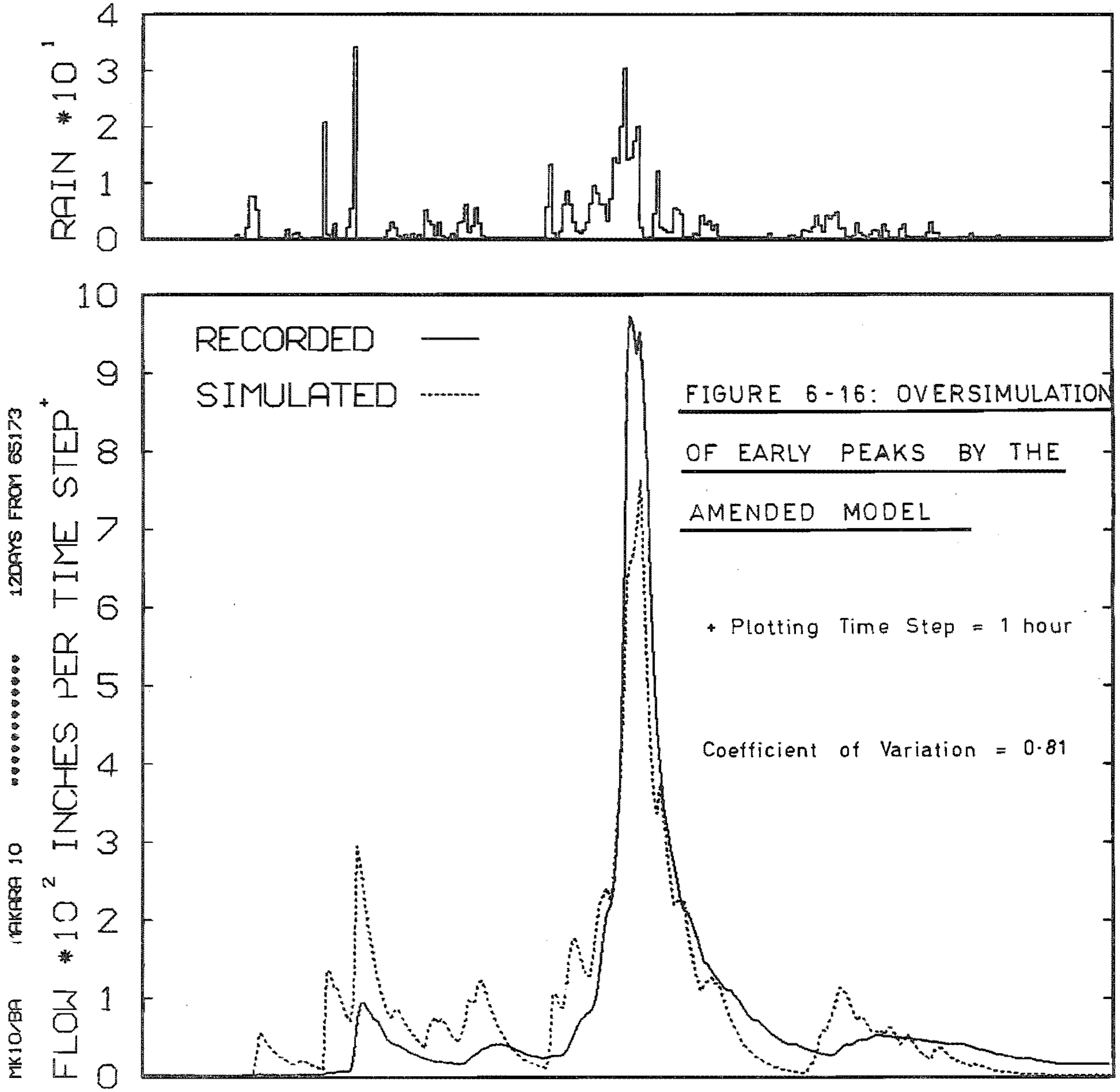

FIGURE 6-16: OVERSIMULATION OF EARLY PEAKS BY THE AMENDED MODEL

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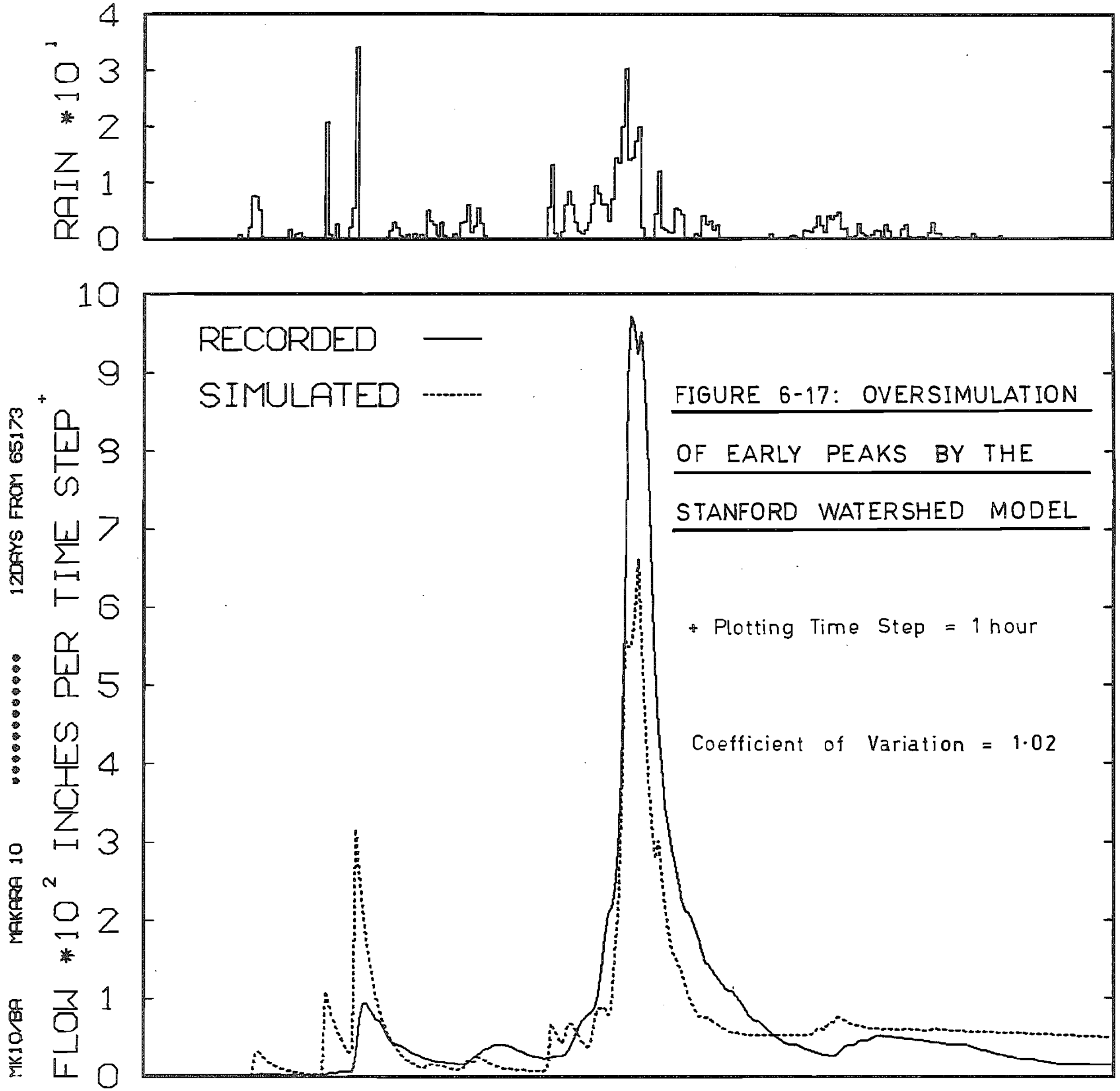

FIGURE 6-17: OVERSIMULATION OF EARLY PEAKS BY THE STANFORD WATERSHED MODEL

+ Plotting Time Step = 1 hour

Coefficient of Variation = 1·02

2344

manner if the rainfall exceeds the conductivity of the soil, or by a step reduction when the storage in the soil is exhausted (see Figure 5-9). The parameters required for satisfactory simulation meant that rainfall never exceeded conductivity so that only the storage exhaustion mechanism occurred. In these circumstances it is surprising the AM was not significantly worse than the SWM at predicting early peaks.

It would be difficult to vary the response of the AM during a storm unless the fraction of catchment area involved in surface flow could be a variable. This would enable the model output to progress, at a rate depending on the model parameters, from the slower-responding subsurface mode to the faster surface mechanism. This concept views surface flow as a distributed action (which it certainly is) but it would be impossible to link this to a lumped soil column solution without violating continuity. Linking variable surface flow to a <u>two</u>-dimensional catchment slice solution is quite possible, and this is discussed in the next chapter.

(b) <u>Recession Shape</u>

The AM was unable to simulate the falling limb of the hydrograph accurately (see Figures 6-18 and 6-19). Typical recorded riverflow recessions, when plotted on semilog graph paper, exhibit one or more decreases in slope, and the SWM could simulate these by <u>two</u> components with different slopes or rates of recession. The AM recession was also linear on a semilog graph but exhibited only a single slope.

If the single subsurface component is required to simulate a composite recession, a soil column lower boundary outflow which was proportional to another power (instead of the first power) of the water table height may approximate

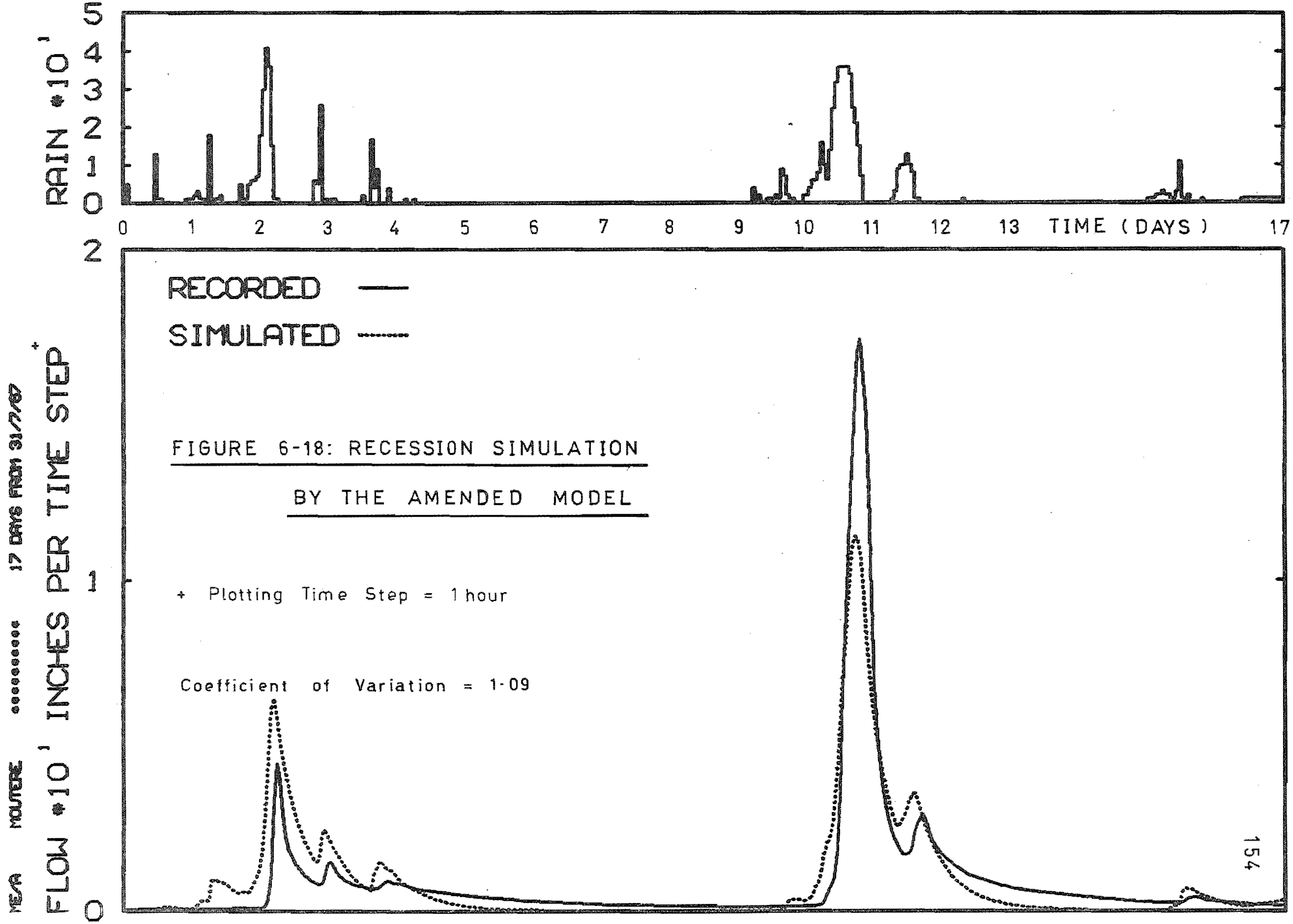

FIGURE 6-18: RECESSION SIMULATION BY THE AMENDED MODEL

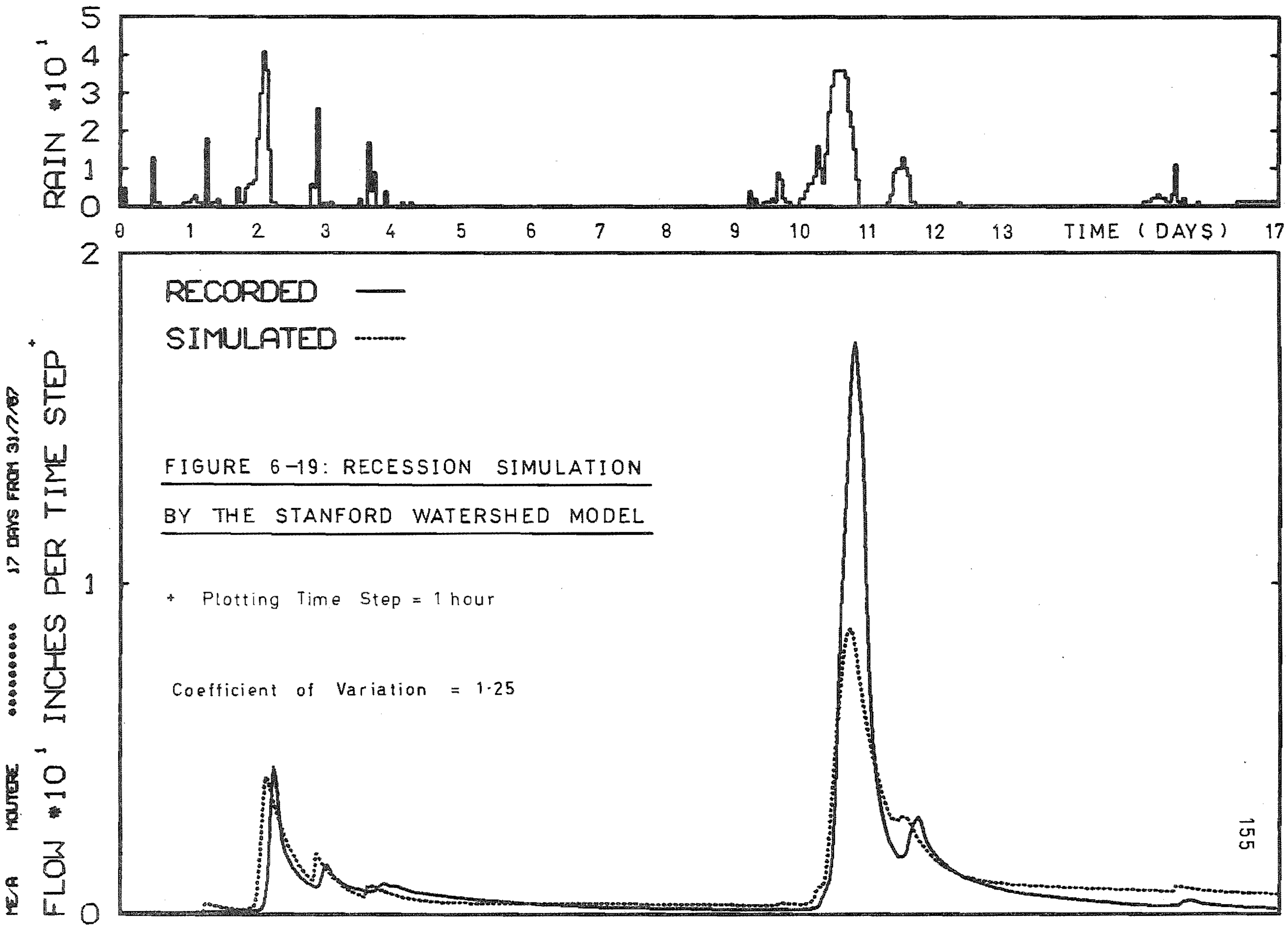

FIGURE 6-19: RECESSION SIMULATION BY THE STANFORD WATERSHED MODEL

reality better. Alternatively a different recession rate could be simulated by a layer of different soil properties at the lower part of the soil column.

(c) Flow Origin

With the parameters chosen for the best fit on the Group 1 records the AM simulated surface flow on only one record. This was a consequence of the "all-or-nothing" nature of the surface flow component which could not allow only part of the catchment to take part in surface flow, and hence produced simulated flows that were too high whenever it acted. Because of the emphasis on peak agreement in this study the fitting process chose parameter values that discouraged surface flow. This meant that in effect most of the records were simulated with a one-component model.

The response of the SWM was simulated mainly by the interflow component as measured by total volume for the records. Surface flow was more frequent than in the AM because the water supply to the land surface did not have to exceed a threshold before surface flow would commence. But because of its short duration the volume of surface flow was generally less than the volume of groundwater, which in turn was less than the volume of interflow.

It is likely that in reality there was surface flow on a varying proportion of the catchments in each storm. This behaviour can only be approximated by the lumped models used here.

(d) Initial Conditions

The assumptions made concerning the moisture status at the start of each simulation, that the storages would be dry, proved to be reasonable. Although this caused the simulated flow to be zero until sufficient rain fell there was no evidence that early parts of the hydrographs were under-simulated because of water satisfying storage in the models. Rather the opposite was the case, as explained in (a) above. Only on a few records with wet antecedent conditions was there a significant effect, and in these cases the simulated flows were low throughout the record, although the fluctuations were correctly predicted. The manual control of the parameter adjustment procedure enabled this to be allowed for.

(e) Amended Model Parameter Values

X(6): Saturated Conductivity Factor, and X(10): Soil Column Outflow Proportionality Constant

Because of their similar effects on the simulations, only X(10) was varied, leaving X(6) at an arbitrarily-chosen value which gave a conductivity of 5.3 inches per hour. For all three catchments the optimum value of X(10) was 10%, which means that the maximum subsurface flow at the bottom of the soil column was 0.53 inches per hour.

Now that a value has been found for the subsurface flow depth via parameter X(12), equation 5-4 may be used to calculate the proportionality constant if the ratio of horizontal to vertical conductivity can be assumed. Using the maximum value for this ratio given by de Wiest(49) which was 30, and the values for X(12) given in Table 6-12, revised values for the proportionality constants were calculated and are shown in Table 6-15.

Now since the conductivity has the same effect on the simulation as the proportionality constant, revised values of conductivity may be calculated to correspond to the revised values of the proportionality constants. These

Table 6-15: Revised Values of Conductivity and Proportionality Constant

Catchment	Revised Proportionality Constant	Revised Conductivity
Makara-10	7.5%	7.1in/hr
Reynolds	1.0%	53in/hr
Moutere	.25%	210in/hr

values, also shown in Table 6-15, are all in the sand to gravel range. Inspection of the three catchments shows that the topsoils consist of material finer than "sand to gravel", and hence would be expected to have lower conductivities. Nevertheless it is possible that the part-surface, part-subsurface process that comprises the rainfall-riverflow path, when modelled by a subsurface flow equation, has an "equivalent conductivity" in the sand to gravel range.

X(11): <u>Subsurface Flow Leakage Fraction</u>

Values for this parameter are all approximately 0.5. However because the response for most of the storms is simulated entirely by the subsurface component, this parameter is not balancing long-term volumes as intended but is balancing storm volumes, and even affecting peak rates.

It is thought extremely unlikely that half the outflow from all three catchments is bypassing the flow-measuring point. Rather it is more likely that half the catchment is not taking part in the flow-generation process, and that rain falling on this area is stored to contribute to baseflow or evaporation later. This view is consistent with the too-rapid recession noted earlier, which exhausts the model of water soon after a storm.

X(12): Soil Column Depth Factor

Values for this parameter varied from 0.8 to 1.0 inches, which correspond to column heights of 20 to 25 inches. Although this seems too large for the depth usually assumed to control catchment response (a few inches only), in this model this dimension must also contain some measure of the horizontal distance from the location of the "average" catchment column to the outlet. Moreover these depth values were obtained using a constant saturated soil moisture content of 0.58. Any difference between the actual soil moisture content and 0.58 would require a compensating adjustment in the soil depth in the model. These depths therefore seem to be reasonable values.

(f) Stanford Watershed Model Parameter Values

x_2: Infiltration Factor and x_3: Interflow Ratio

Values of x_2 ranged from 0.25 to 0.5 inches per hour, and values of x_3 from 3 to 4. These figures correspond to a catchment average infiltration rate of 0.12 to 0.25 inches per hour and average interflow supply of 0.25 to 1.0 inches per hour, when the model storages are dry at the start of a storm. The model reduces these rates as the storages fill during the course of a storm.

The infiltration factors are at the low end of the range given for this parameter by Crawford and Linsley[17] (0.3 to 1.2 inches per hour), while the interflow ratios are higher than expected (0.5 to 3.0).

x_6: Daily Interflow Recession Constant, and x_7: Daily Groundwater Recession Constant

The interflow constant values varied from .01 to 0.1, while the groundwater constant was found to be 0.8 for all catchments. The interflow values were unexpectedly low, but because of the large contribution to the simulated riverflow by interflow this parameter was called upon to influence the peak flows. The groundwater value is also lower than expected, but because only a few days before and after each storm were simulated the values could not be determined reliably.

Crawford and Linsley[17] state that the recession constants can be obtained by recorded hydrograph analysis. This is not so for the groundwater component because of storage of water in other parts of the model which continues to feed the groundwater storage even in the absence of rainfall. Therefore the simulated groundwater recession is slower than that given by the specified recession constant. This may explain the lower value than expected found for x_7.

x_9: Groundwater Leakage Fraction

The values of 0.5 for this parameter imply a small amount of flow bypassing the catchment outlet, since the groundwater forms a small part of the total simulated volume. The values are therefore reasonable.

(g) General

The parameter values found above by trial-and-error were those which could not reliably be estimated by any other means. In view of the satisfactory performance of the AM described in this chapter, and the potential of more general formulations of Richards' equation outlined in the next chapter, there is an urgent need for the widespread measurement of the hydraulic soil properties which could enable the prediction of these parameters in advance.

CHAPTER SEVEN

DEVELOPMENT

7.1 More General Formulations of Richards' Equation

A final step remains before the aim of this study is fulfilled. Having shown that the replacement of the subsurface components of the SWM by a one-dimensional solution to Richards' equation results in a model which performs satisfactorily, the potential of more general formulations of the equation for effecting improved model performance must now be estimated. This chapter does this by examining the behaviour of published numerical solutions to the two-dimensional form of Richards' equation with a view to incorporating them in a catchment model. In addition the increased demands for information about the catchment made by increasingly detailed models leads to some suggestions for future data collection requirements, particularly concerning the hydraulic properties of the soil.

A. The Work of Freeze

Although possible in principle, solution of Richards' equation in three dimensions is at present beyond the capacity of most computers. There is also a scarcity of detailed information on the soil properties required by three-dimensional models. Therefore we look in this section at two-dimensional solutions.

The solutions most relevant to catchment modelling were obtained by Freeze. Freeze first solved Richards' equation for a two-dimensional catchment slice such as shown in Figure 4-5, which was connected to a constant-head stream with a possible adjacent seepage face[29]. The solution, as for his one-dimensional solutions, took the form of the distribution of Ψ ; this distribution changed with time in response to the imposed boundary conditions. From this the water table position was obtained as the line where $\Psi=0$, and fluctuations of the water table resulting from various hypothetical combinations of rainfall and soil properties were presented (see Figure 7-1). Most importantly, the solution for a heavy rainfall predicted a growing saturated zone extending outwards from the channel. It is this ability to predict a growing saturated area, on which all subsequent rainfall would contribute to surface flow, which is the most valuable feature of this solution.

In a later paper[50] Freeze connected his two-dimensional slice solution to a solution for flow in a channel, with output from the subsurface zone and saturated surface area forming the lateral input to the channel. The simulations carried out with this model (again using artificial data) indicated that storm riverflow could rarely be simulated by a wholly-subsurface flow solution. Only where a convex hillside feeds a deeply-incised channel and when conductivity is very high can subsurface flow be significant. It will be recalled that high conductivity was also required to simulate storm riverflow using real data with the one-dimensional solution of the AM.

Nevertheless Freeze was able to simulate realistic hydrographs by assuming that rain falling on the growing saturated area adjacent to the channel became riverflow with no lag, and that this flow formed the major part of the

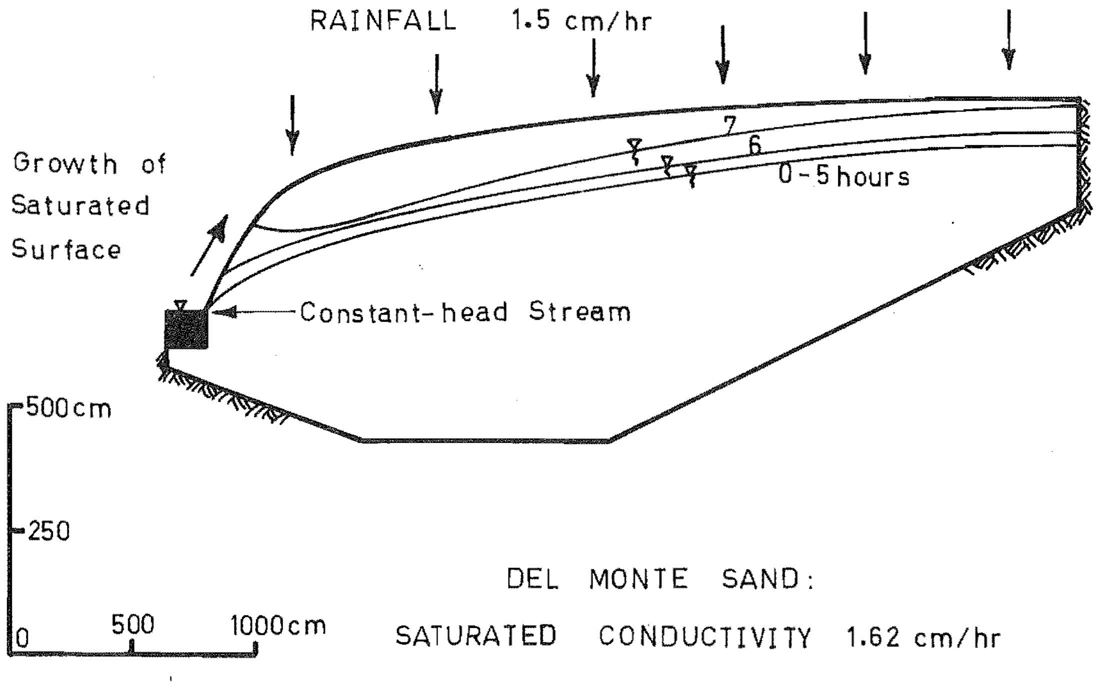

FIGURE 7-1: GROWTH OF SATURATED SURFACE PREDICTED BY FREEZE'S TWO-DIMENSIONAL SOLUTION

response. Saturation occurred on this area via a water table rising from below rather than by the classical exceedance of the conductivity by the rainfall intensity; again this is similar to the mode of operation of the one-dimensional solution in the AM. Thus the major role of the subsurface zone is seen as that of predicting the variable surface area contributing to prompt riverflow, for which at least a two-dimensional solution will be required.

B. Re-examination of Expected Advantages

The expected advantages of a Richards' equation solution description of subsurface flow in a catchment model, originally given in Section 3.5, are now re-examined in the light of the experience with the AM and the capabilities of Freeze's two-dimensional solutions.

(a) Physical Reality

Even if a catchment displayed complete areal homogeneity of soil properties the response to a uniform rainfall would cause differing water table rises at different locations because of lateral flow caused by catchment slope. This questions the validity of the "typical" soil column used in the AM in attempting to model the subsurface zone as a whole. The correspondence of Freeze's two-dimensional solution to an actual three-dimensional catchment is much more plausible.

(b) Partial Area Runoff

A deficiency in the behaviour of the AM was the step-function reduction in the infiltration rate, when the rising water table reached the surface accompanied by the catchment-wide commencement of surface flow. This action, necessitated by the one-dimensional nature of the soil column formulation, highly over-simulated the peak flows. The parameter fitting process was then forced to suppress the occurrence of surface flow to improve agreement with the recorded riverflows, by adopting very large values of conductivity.

There is growing field evidence[3,4] that surface flow occurs on only a small fraction of the area of many catchments, especially in humid climates. The term "loss" in these cases may be used to describe the rain falling on the unsaturated area of the catchment where it avoids becoming surface flow, rather than a catchment-wide value of infiltration exceeded by the rainfall. Just as the increasing response of a catchment during a storm may be explained by an infiltration capacity decreasing with time, so can it be explained by an increasing saturated area on which rain falling becomes riverflow with very little lag.

The decrease of unsaturated area with time during a storm is able to be simulated by Freeze's two-dimensional solution (see Figure 7-1). A rainfall of less than the saturated conductivity caused the saturated area to grow outward from the channel at a rate which would be controlled by the geometry of the catchment slice, the soil properties and the initial moisture conditions. This control could enable the model-builder to overcome the high simulations of early peaks in a storm noted with both the AM and the SWM. Further, by enabling the rapidly-responding surface flow component to be brought into action gradually, a two-dimensional soil slice model would avoid the artificially large conductivity values required for satisfactory simulation by the AM.

(c) Component Processes

In combining the functions of three SWM components (infiltration, interflow and groundwater) the soil column solution in the AM suffered from a lack of flexibility and was unable to simulate recessions accurately. A two-dimensional solution could improve the recession simulation in two ways. Firstly the soil solution will no longer be attempting to simulate the whole hydrograph, as the AM was forced to do on most of the storms because of the suppression of surface flow by the parameter choice. The more appropriate conductivities will give a slower subsurface response which should better fit the recessions. Secondly the opportunity to specify a layered soil will give the freedom to simulate a variety of recession shapes. The action of layers is to promote lateral flow and this would be difficult to incorporate into a one-dimensional model.

(d) Areal Variation

A two-dimensional solution to Richards' equation could handle both vertical and horizontal inhomogeneity in the soil properties. An example is the simulation of a perched water table by Freeze's solution, shown in Figure 7-2. Where the required data about the soil is available the two-dimensional solution avoids the one-dimensional restriction of using average soil properties.

(e) Parameter Estimation

Some difficulty was experienced in the AM in evaluating the soil column outflow proportionality constant. The corresponding boundary condition in a two-dimensional solution would be specified via a water level at the channel boundary consistent with conditions in the channel, and a seepage face above this level. Freeze[36] found that

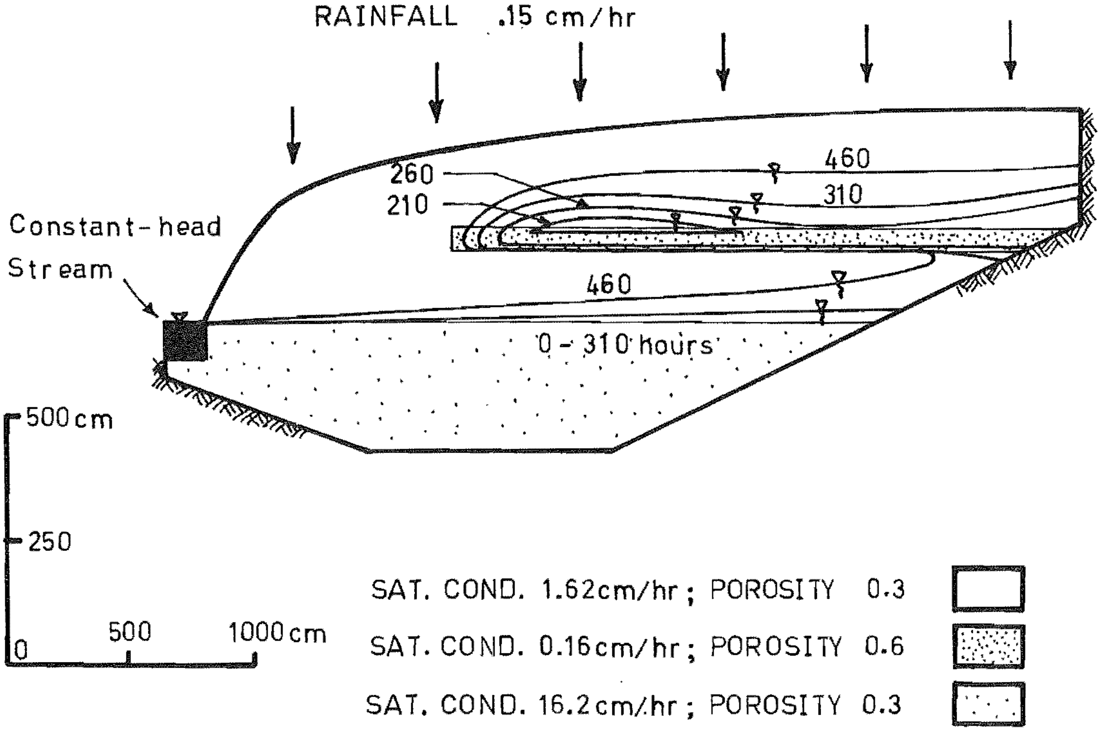

FIGURE 7-2: FREEZE'S SIMULATION OF A PERCHED WATER TABLE

coupling a channel-flow solution to a two-dimensional soil slice solution was relatively straightforward; using the stream depth from the previous time step as an initial guess for the common boundary condition, he seldom had to employ more than one iteration to obtain convergence. He concluded that at least for wide streams with a small depth variation no iteration was necessary; knowledge of the stage-discharge relation for the stream will therefore allow this boundary condition to be evaluated without arbitrary constants.

The values of the other soil parameters which are required to ensure that the model duplicates the prototype behaviour will correspond more closely to those measured in the field for the more realistic two-dimensional solution.

C. Evaluation

Any improvement in model realism is certain to incur increased complexity. Both the computer storage and the calculation time required for a more complex solution will rise roughly in proportion to the number of nodes by which the space dimensions of the system are represented. The one-dimensional solution in the AM used 25 nodes to represent the column height, so an extension to two dimensions might increase storage and time needed by a factor of 25.

This could be accommodated on the IBM 360/44 computer used for the simulations in this study (128K bytes of core storage) at the expense of the size of the arrays which enabled general, time-varying input to be specified. The time required for a 200-time-step simulation would rise from about half a minute to twelve minutes. Freeze reported that an IBM 360/91 with 1500K bytes of core storage could handle 30,000 nodes and simulate a 2500-node, 200-time-step solution in five to ten minutes. It seems that a two-dimensional solution could be handled by a medium-sized computer with some difficulty, or by a large computer with ease.

An additional problem which will accompany a more general description of the subsurface part of the catchment is that the solution to the surface flow equations will have to be more general. Many surface flow solutions (including that in the SWM) assume that the input is distributed uniformly in space, which implies that a constant length or area is taking part. The surface solution must be capable of recognising that the area involved will increase and decrease as the storm runs its course.

The surface flow solution will also have to be coupled to the subsurface solution so that continuity across the common boundary is maintained. This was achieved in the one-dimensional solution used here by iterating on the common boundary condition (the infiltration rate) to obtain convergence of the pressure at the ground surface predicted by the two solutions. In a two-dimensional case this iteration would be very lengthy since there will be a whole series of common boundary conditions to find. Thus the time of twelve minutes suggested above may be too small.

The two-dimensional catchment slice as a model for the subsurface zone of a catchment has been shown to be superior to the one-dimensional solution used in the AM, which was found to give satisfactory performance in this study. Most of the technology required for using the catchment slice in a catchment model exists[36,50] and only some details of adapting the surface flow description to deal with partial-area surface flow require development. It is therefore concluded that a model employing a two-dimensional catchment slice solution to Richards' equation could perform better than both the AM and the SWM, provided that more detailed knowledge of the soil properties is available.

7.2 Data Requirements

More detailed models require more detailed information about the prototype. When collecting data for this study it was found that suitable rainfall and riverflow data for peak simulation on small (and large) catchments were not widespread, and that measurements of the hydraulic properties of soils were even less common. The sources of data for models such as used herein are now discussed.

A. Soil Properties

The commonly measured properties of soils in New Zealand generally emphasise the agricultural or mechanical qualities rather than the hydraulic behaviour. The following list typifies the available data:

(a) Determination for 54 soil types throughout New Zealand of bulk density, porosity and moisture contents corresponding to suctions of 0.2 and 15 atmospheres (referred to as the field capacity and wilting point respectively). Several measurements of each soil type were made at various depths down to 3 feet (1m) and the results published by the Department of Scientific and Industrial Research[44].

(b) Maps of the distribution of major soil groups of New Zealand, to a scale of 1:1,000,000, also published by the DSIR[51].

(c) Measurement of the moisture content-soil suction relation for major soil groups of the North Island of New Zealand[52] and for occasional South Island soils[53].

(d) Soil moisture measurements by gravimetric and neutron scatter methods on some experimental basins[54] and irrigation research stations[53].

(e) Infiltration capacity using infiltrometer measurements by various agencies, including the Universities[55] and the Ministry of Works[56].

(f) Ad hoc measurements of the mechanical properties of soils by civil engineering agencies concerned with earth structures and building foundations.

The salient omission is that of the hydraulic conductivity-soil suction relation and even of the saturated hydraulic conductivity, although it is possible that these can be arrived at indirectly by fitting a Richards' equation solution to the infiltrometer situation. The measurement for representative soils of the conductivity-suction relation and the extension to soils in the South Island of the moisture content-suction measurements is required before the application of more realistic subsurface flow descriptions to catchment models can proceed. Also required for modelling particular catchments will be the depths to impermeable parent material, especially for the near-channel areas which the partial-area concept postulates will control the prompt response.

B. Rainfall and Riverflow Data

This study also demonstrated the scarcity of catchments in New Zealand with high-quality rainfall and riverflow records. The few catchments with long records do not have data at sufficiently small time intervals to adequately define the peak flows, or else have insufficient raingauges to adequately define the areal variations of rainfall. Since 1965 under the Representative Basin Program of the Ministry of Works[42] instrumentation of a representative catchment in more than half of the 90 hydrological regions of New Zealand has been achieved, and data from these are generally good. When these catchments have been operating for longer the data situation will be much improved.

However the storage of this data, at present divided among several authorities, requires to be rationalised. A central databank, perhaps computerised, should contain all rainfall and riverflow records from representative and experimental catchments. This would allow the user to obtain all his data from a single source and without having to repeat the basic chart-reading procedures. The bank could be patterned after the recommendations of Ibbitt[7], who set out requirements for "standard data sets" for testing conceptual catchment models. The data would necessarily be accompanied by appropriate information about the catchment of origin.

CHAPTER EIGHT

CONCLUSION

8.1 Past

The review of mathematical catchment models in Chapter Two showed that few were sufficiently comprehensive to simulate the complete land phase of the hydrologic cycle from precipitation to riverflow without separate "loss" and "baseflow" calculations. The Stanford Watershed Model stood out as a "complete" model, and additionally as one with a structure capable of accepting revised descriptions of the component processes.

Examination of Richards' equation for flow of water in a saturated or unsaturated porous medium revealed that the knowledge existed for replacing the infiltration and subsurface flow components of the Stanford Watershed Model with a treatment based on physical laws. Boundary conditions for this equation appropriate for the catchment situation were given. It was desired to determine whether this replacement would allow improved simulation of peak flows on small catchments.

8.2 Present

It is concluded that the description of the subsurface zone of a catchment by Richards' equation <u>will</u> improve the ability of a mathematical model to simulate peak flows on hydrologically small catchments, provided that the solution is carried out in two or more dimensions.

This conclusion was based on the testing of a version of the Stanford Watershed Model in which the subsurface components had been replaced by a Richards' equation solution for a one-dimensional vertical column of soil, as a pilot study for a more detailed treatment. The performance of this Amended Model was comparable to that of the Stanford Model, while an examination of Richards' equation solutions to two-dimensional slices of soil revealed several areas where significant improvement over the one-dimensional solution might be made.

During the course of the study several other conclusions were drawn:

(a) The one-dimensional form of Richards' equation can be used as a component in a digital catchment model. Boundary conditions can be imposed which maintain continuity with surface flow at the ground surface, and describe an input to riverflow at the lower end.

(b) Suitable calculation time steps for peak models can be obtained by considering the local curvature of the peaks of the recorded riverflow hydrographs. Values thus found were confirmed by considering the decrease in unitgraph peak with increase in rainfall excess duration, and found to be satisfactory by use with both the Stanford and Amended Models.

(c) Because of the computer time requirements of automatic methods, and because no single index of performance can adequately describe the fit between two hydrographs, manual parameter adjustment using simultaneous plotting of recorded and simulated hydrographs is superior to automatic optimisation of a numerical index.

(d) The performance of the Amended Model on three catchments in New Zealand using a split-record test was not significantly different from that of the Stanford Model. In addition the Amended Model had one less parameter which required trial-and-error evaluation. Both models had insufficient provision for a reduced response early in a storm and the Amended Model generally under-simulated the recessions.

(e) Values of the four parameters of the Amended Model which had to be found by fitting model output to recorded riverflow attained physically realistic values, except for the hydraulic conductivity. In order to compensate for the lack of simulated surface flow the conductivity had to assume values in the sand-to-gravel range for satisfactory simulation. This was caused by the inability of the model to simulate surface flow on only part of the catchment, which caused the fitting process to discourage surface flow altogether.

(f) Values of the five parameters of the Stanford Model which had to be found by fitting also attained realistic values, consistent with the recommendations of its model-builders. There was little variation between catchments for the fitted parameters for either model.

(g) Solution of Richards' equation for a two-dimensional slice of soil would entail a 25-fold increase in the computer storage and calculation time required. However, besides being a closer approximation to the real, three-dimensional catchment, such a solution could eliminate one of the parameters which has to be fitted, allow for catchment inhomogeneity and simulate partial-area surface runoff.

8.3 Future

Extension of the Amended Model described herein to include a two-dimensional Richards' equation solution to describe the subsurface zone of the catchment is encouraged by these conclusions. The performance of such an extended model, provided that detailed knowledge of the soil properties was available, would provide a valuable test of the applicability of the partial-area runoff concept, which in turn could have vital implications for studies of the effects of land-use changes.

Measurement of the hydraulic properties of representative soils in New Zealand is needed to provide basic data for this type of model. Centralised storage of rainfall and riverflow data from New Zealand's representative and experimental catchments would assist the users of all types of catchment model.

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APPENDIX A

DESCRIPTION OF AMENDED MODEL COMPUTER PROGRAM

A-1 General

This program translates catchment average rainfall, input at regular intervals of time which may be any submultiple of a day, and daily potential evapotranspiration into riverflow for a catchment whose characteristics are input as values of 15 parameters. The catchment must be small enough so that travel time of water in the channel network is negligible, and there must be no snow during the periods simulated.

The program was written in FORTRAN IV for the IBM 360/44 at the University of Canterbury, Christchurch, New Zealand. With a core storage of 128K bytes this computer can simulate an hourly record up to three months long, with a proportional reduction when time steps of less than one hour are used. Calculation time (excluding input and output) is seven seconds per day for hourly simulation when no time step increase is allowed during low-flow periods. When the time step was allowed to increase by up to 24 times the saving in time ranged up to 75%, depending on the proportion of high and low flows in the records.

A-2 Input

Input to the program may be classified into four groups:

(a) Control Parameters

These define the length of the record to be simulated, the time step to be used, the formats of the rainfall, evapotranspiration and recorded riverflow, the options available during calculation and the type of output required.

(b) Catchment Parameters

These specify the average values of the vegetation cover, the catchment surface parameters and the soil properties.

(c) Initial Conditions

These are the amounts and distribution of water in the model storages representing interception, surface flow and the subsurface zone at the start of the period to be simulated.

(d) Meteorological Conditions

These are the volumes of rainfall in each interval of the period and the volumes of potential evapotranspiration in each day of the period to be simulated. If the corresponding recorded flows are known, they are also input as volumes for each time interval in order to evaluate the performance of the simulation.

A complete list of the input requirements appears in Table A-1.

A-3 Output

Although most of the following items may be suppressed the maximum output consists of:

(a) Identification of the program, catchment and record.
(b) Echo of all control and catchment parameters.
(c) The moisture distribution each time step of the simulation. This is usually suppressed.
(d) A combined graph of rainfall, recorded flow and

simulated flow versus time, on the computer line-printer.

(e) Cards punched for input to an off-line Calcomp Plotter of a graph similar to (d). Either (d) or (e) is normally suppressed.

(f) A summary of performance consisting of six numerical indices of fit between the recorded and simulated flows (see Table 6-8), the calculation time, the percentages of the total simulated flow volume contributed by the different model components, and a check of the water balance over the period.

(g) The initial and final distributions of water in the storages of the model.

A sample output, without items (c), (d) and (e), is shown in Figure A-1. The listing of the program appears in Figure A-2.

Table A-1: Input Requirements of the Amended Model Computer Program

Card	Format	Variable	Description
CONTROL PARAMETERS			
1	80A1	FABEL	Catchment, Record and comments
2	5A4	FMTP	Rainfall input format
	5A4	FMTE	Potential Evapotranspiration input format
	5A4	FMTF	Recorded Riverflow input format
3	I5	ND	Number of Days to be simulated
	I5	K	Number of Time Steps per day
	I5	MS	Number of Time Steps per graph ordinate
	I5	LH	Not used
	I5	NDUM	Number of Time Steps Ignored when calculating performance index
	I5	NPOW	0=Initial Conditions given by array of pressure or suction 1=by Moisture Deficit to saturation 2=by Water Table Height as a fraction of column height
	I5	NPU	1=Graph on Line-printer 0=No graph
	I5	NPA	1=Graph on Calcomp Plotter 0=No graph
	F5.0	QMAX	Full-page Ordinate for flow graph
	F5.0	PMAX	Full-page Ordinate for rainfall graph. Both QMAX and PMAX are automatically increased if too small
	I5	NPCH	2=Punch Simulated Flows onto cards in format FMTF 11=Punch Final Moisture conditions onto cards in format 5F5.2/16F5.2 0=Do neither
	I5	NDZ	Number of Nodes to represent distance dimension in soil column numerical solution
	I5	NP	Number of Time Steps per full moisture distribution printout
	I5	NUP	Maximum Number of Time Steps combined in low-flow periods
	F5.0	TUP	Rainfall Threshold to prevent any time step increase
	F5.0	TDN	Not used
4	F5.0	ACARD	Not used

Continued Over

Table A-1 continued: Input Requirements of the Amended Model Computer Program

Card	Format	Variable	Description
	CATCHMENT PARAMETERS		
5	10X, F10.0	X(13)	Manning's "n" for surface flow
	F10.0	X(14)	Maximum surface flow distance, ft
	F10.0	X(15)	Average surface slope
6	F10.0	X(1)	Moisture content-soil suction shape factor A (see equation 5-20), inches of water
	F10.0	X(2)	Moisture content-soil suction shape factor B (see equation 5-20), inches of water
	F10.0	X(3)	Saturated Moisture Content factor (see Table 5-1)
	F10.0	X(4)	Hydraulic conductivity-soil suction shape factor A (see equation 5-20), inches of water
	F10.0	X(5)	Hydraulic conductivity-soil suction shape factor B (see equation 5-20), inches of water
	F10.0	X(6)	Saturated Conductivity factor (see Table 5-1), (inches per hour)$^{-1}$
7	F5.0	X(7)	Average Interception capacity, inches of water
	F5.0	X(8)	Maximum Evaporation rate, inches per day
	F5.0	X(9)	Threshold Moisture Content at which evapotranspiration ceases, as a fraction of saturated moisture content
	F5.0	X(10)	Soil Column Base Outflow Proportionality Constant, percent
	F5.0	X(11)	Subsurface Flow Leakage Fraction
	F5.0	X(12)	Soil Column Depth Factor, inches per node

Continued Over

Table A-1 continued: Input Requirements of the Amended Model Computer Program

Card	Format	Variable	Description
INITIAL CONDITIONS			
8	F5.2	EXPI	Initial Intercepted Water, inches
	F5.2	DELDI	Initial Surface Water Increase Rate, inches per hour
	F5.2	DII	Initial Surface Water Depth, inches
	F5.2	SDI	If NPOW=1, Initial Moisture Deficit to saturation, inches. Otherwise not used
	F5.2	HXI	If NPOW=2, Initial Water Table Height, as a fraction of column height Otherwise not used
9	16F5.2	PIN	If NPOW=0, Initial Array of Suction or Pressure, inches of water Otherwise omit
METEOROLOGICAL CONDITIONS			
10ff	FMTP	PRECI	Array of Rainfall Volumes, in inches, one per time step
11ff	FMTE	EPI	Array of Potential Evapotranspiration volumes, in inches, one per day
12ff	FMTF	QR	Array of Recorded Riverflow Volumes, in inches, one per time step

```
FALTER WATERSHED MODEL

MK10/FB      MAKARA 10      ***************      7DAYS FROM 63217

CONTROL PARAMETERS
  DT= 1.00    AC=.0010    TUP= 0.0    TDN= 0.0      K=   24    NK=    0    ND=    7   NDZ=   26    MS=    1  NDUM=    0    NP= 9999
NPOW=    2  NPCH=    0    NUP=    1   NPU=

CATCHMENT CHARACTERISTICS
A1=   13.80   A2=    27.60   A3=  200.00   B1=    19.70   B2=      7.88   B3=   30.00
EPXM=  0.10   EMAX=   0.50   THWP=   0.0   XK24= 10.00    XK24L= 0.50     DZ=   0.80    EN=        0.300   EL=     390.00   ES=       0.580

SATURATED MOISTURE CONTENT =  0.58    SATURATED PERMEABILITY =  5.27    SOIL DEPTH = 20.00

PERFORMANCE SUMMARY


SUM OF THE ONETHS   -0.560      SUM OF THE SQUARES  0.017583      SUM OF THE FOURTHS 0.000013429

COEFT CORRELATION    0.971      COEFT OF VARIATION     0.575      COEF DETERMINATION        0.958

CALC TIME WAS   46  SECONDS          TOTAL RECORDED FLOW       2.991

WATER BALANCE                                                                          FLOW ORIGIN IN PERCENT

IN                                          OUT

TOTAL RAINFALL              5.221       TOTAL OUTFLOW              2.432                  RUNOFF      0.0

DECREASE IN STORAGE        -0.018       TOTAL EVAPOTRANS           0.192                 THRUFLOW   100.0

                                        GROUNDWATER LOSS           2.432                             ------

                       ----------                             ----------                             100.0

                            5.203                                  5.055

MOISTURE STATUS     AT T =      0     EPX    0.0       DELD    0.0       DI    0.0       SMOIST    1.902

MOISTURE STATUS     AT T =    168     EPX    0.018     DELD    0.0       DI    0.0       SMOIST    1.902

PRESSURE AT T =     0      1      0.0        2    -0.800      3    -1.600      4    -2.400      5    -3.200      6    -4.000      7    -4.800      8    -5.600
                           9     -6.400     10    -7.200     11    -8.000     12    -8.800     13    -9.600     14   -10.400     15   -11.200     16   -12.000
                          17    -12.800     18   -13.600     19   -14.400     20   -15.200     21   -16.000     22   -16.800     23   -17.600     24   -18.400
                          25    -19.200     26   -20.000

MOISTURE AT T =     0      1      0.583      2     0.576      3     0.569      4     0.562      5     0.555      6     0.548      7     0.540      8     0.533
                           9      0.525     10     0.517     11     0.509     12     0.501     13     0.493     14     0.485     15     0.477     16     0.469
                          17      0.460     18     0.452     19     0.444     20     0.435     21     0.427     22     0.419     23     0.411     24     0.403
                          25      0.395     26     0.387

PRESSURE AT T =   168      1      0.000      2    -0.800      3    -1.600      4    -2.400      5    -3.200      6    -4.000      7    -4.800      8    -5.600
                           9     -6.400     10    -7.200     11    -8.000     12    -8.800     13    -9.600     14   -10.400     15   -11.200     16   -12.000
                          17    -12.800     18   -13.600     19   -14.400     20   -15.200     21   -16.000     22   -16.800     23   -17.600     24   -18.400
                          25    -19.200     26   -20.000

MOISTURE AT T =   168      1      0.583      2     0.576      3     0.569      4     0.562      5     0.555      6     0.548      7     0.540      8     0.533
                           9      0.525     10     0.517     11     0.509     12     0.501     13     0.493     14     0.485     15     0.477     16     0.469
                          17      0.460     18     0.452     19     0.444     20     0.435     21     0.427     22     0.419     23     0.411     24     0.403
                          25      0.395     26     0.387

  STOP        0
/ε
```

FIGURE A-1: OUTPUT FROM THE AMENDED MODEL COMPUTER PROGRAM

```
      CIMENSICN X(15)
      CALL FALTER(SLM,1,X)
      ENC


      SLBRCLTINE FALTER(SCARE,INCEX,X)
C
C     CIGITAL WATERSHEC MCCEL WITH FREEZE-TYPE INFILTRATICN ANC
C     PLANE-TYPE OVERLANC FLCW
C
C     INPLT ANC SETLP STAGE
C
      CIMENSICN X(15),TH(100),FMTP(5),FMTE(5),FMTF(5),P(1CC),THIN(1CC),F
     1IN(10C),PLL(1CC)
      DIMENSICN PRECI(22C8),C(22C8),QR(22C8),EPI(366)
      CCLBLE PRECISICN XK,CC
      LCGICAL*1 LINE(132)/132*'.'/,FABEL(8C)
      SLMC=C.
      TCUT=C.
      KP=C
      TGWF=C.
      TBILL=C.
      TEP=0.
      GLCSS=C.
      IF(INCEX.NE.1) GC TC 4
      READ(5,2) FABEL,FMTP,FMTE,FMTF
    2 FCRMAT(80A1/15A4)
      READ(5,6) NC,K,MS,LF,NCLM,NPCW,NPL,NPA,CMAX,FMAX,NPCF,NCZ,NF,NLF,T
     1LP,TCN
    6 FCRMAT(8I5,2F5.C,4I5,2F5.C)
      DT=24./FLCAT(K)
      AC=.001
      NCT=NC*K
      READ(5,23) ACARC
   23 FCRMAT(16F5.C)
      READ(5,26) X(13),X(14),X(15)
   26 FCRMAT(10X,7F10.0)
      READ(5,1) (X(I),I=1,12)
    1 FCRMAT(6F10.0/6F5.C)
      READ(5,46) EPXI,CELCI,CII,SCI,FXI
      IF(NPCW.EC.1) READ(5,46) (FIN(J2),J2=1,NCZ)
   46 FCRMAT(16F5.2)
      READ(5,FMTP) (PRECI(J1),J1=1,NCT)
      READ(5,FMTE) (EPI(J1),J1=1,NC )
      READ(5,FMTF) (QR(J1),J1=1,NCT)
    4 CZ=X(12)
      IF(INCEX.NE.1.ANC.INCEX.NE.11) GC TC 5
      WRITE(6,25)
   25 FCRMAT('1')
      WRITE(6,36) LINE
      WRITE(6,3)
    3 FCRMAT('OFALTER WATERSHEC MCCEL')
      WRITE(6,29) FABEL
   29 FCRMAT(/'C',8CA1)
      WRITE(6,36) LINE
      WRITE(6,31)
   31 FCRMAT('CCCNTRCL PARAMETERS')
      WRITE(6,3C) CT,AC,TLP,TCN,K,NK,NC,NCZ,MS,NCLM,NP,NPCW,NPCF,NLP
   30 FCRMAT('    DT=',F5.2,'    AC=',F5.4,'    TLP=',F5.2,'    TCN=',F5.2,
     1'      K=',I5,'      NK=',I5,'      NC=',I5,'   NCZ=',I5,'    MS=',I5,'
     2  NCLM=',I5,'     NP=',I5/' NPCW=',I5,'  NPCF=',I5,'    NLP=',I5,'
     3NPL=',I5,'  NPA=',I5)
      WRITE(6,32)
   32 FCRMAT('OCATCFMENT CHARACTERISTICS')
      WRITE(6,33) X
   33 FCRMAT(' A1=',F8.2,'   A2=',F8.2,'    A3=',F8.2,'    B1=',F8.2,'    B
     12=',F8.2,'    B3=',F8.2/' EPXM=',F6.2,'   EMAX=',F6.2,'   THWP=',F6
     22.2,'   XK24=',F6.2,'   XK24L=',F5.2,'     CZ=',F6.2,'   EN=',F11.3
     33,'   EL=',F10.2,'   ES=',F1C.3)
      EX1=XK(C.,X(1),X(2),X(3))
      EX2=XK(C.,X(4),X(5),X(6))
      EX3=X(12)*FLCAT(NCZ-1)
      WRITE(6,54) EX1,EX2,EX3
   54 FCRMAT('OSATURATEC MCISTURE CCNTENT =',F6.2,'     SATLRATEC FERMEAB
     1ILITY =',F6.2,'     SCIL CEPTH =',F6.2)
      WRITE(6,36) LINE
   36 FCRMAT(' ',132A1)
      SLMCR=0.
      CELC=C.
      SLMP=C.
      PREC=C.
      EP=C.
      CC 9 I=1,NCT
      SLMCR=SLMCR+CR(I)
    9 SLMP=SLMP+PRECI(I)
```

FIGURE A-2: THE AMENDED MODEL COMPUTER PROGRAM

```
      IF(NPOW.NE.2) GO TO 5
      PIN(1)=HXI*EX3
      DO 21 I=2,NDZ
   21 PIN(I)=PIN(I-1)-X(12)
      STEP=DZ*4.
      NN=1
      ERL=0.
    5 TTHIN=0.
      DO 7 J2=1,NDZ
      P  (J2)=PIN(J2)
      THIN(J2)=XK(P  (J2),X(1),X(2),X(3))
    7 IF(J2.GT.1) TTHIN=TTHIN+0.5*(THIN(J2)+THIN(J2-1))*DZ
      IF(NPOW.NE.0) SOI=EX3*EX1-TTHIN
      ER=EX1*EX3-TTHIN-SOI
      IF(NPOW.NE.0.OR.ABS(ER).LT.AC.OR.P(NDZ).LT.-1.E5) GO TO 12
      IF((ER.GT.0..AND.ERL.LT.0.).OR.(ER.LT.0..AND.ERL.GT.0.)) STEP=STEP
     1/4.
      PIN(NDZ)=PIN(NDZ)+ER*STEP/ABS(ER)
      DO 20 I=2,NDZ
   20 PIN(NDZ+1-I)=PIN(NDZ+2-I)+DZ
      ERL=ER
      NN=NN+1
      GO TO 5
   12 EPX=EPXI
      DELC=DELCI
      DI=DII
      DTV=DT
      NC=1
      IT=KLOCK(J)
C
C     HOURLY CALCULATION STAGE
C
      DO 8 I=1,NDT
      Q(I)=0.
      KP=KP+1
      IF(KP.GT.NP) KP=1
      CALL FINDH(P  ,NDZ-1,H2,H3,DZ,0)
      NB=(1.-H3/EX3              )*FLOAT(NLP)
      PREC=PREC+PRECI(I)
      NDAY=(I-1)/K+1
      NHR=I-(NDAY-1)*K
      FK=FLOAT(K)*0.875
      IF(NHR.GT.K/8) EP=EP+EPI(NDAY)*(1.-COS(6.29*FLOAT(NHR-K/8)/FK))/FK
      IF(I.EQ.NDT.OR.NC.GE.NB.OR.I.LT.5) GO TO 38
      IF(PRECI(I+1).GT.TUP.OR.PRECI(I).GT.TLP.OR.PRECI(I-1).GT.TLP.OR.PR
     1ECI(I-2).GT.TLP.OR.PRECI(I-3).GT.TLP.OR.PRECI(I-4).GT.TLP)GO TO 38
      DTV=DTV+DT
      NC=NC+1
      GO TO 8
   38 NR=PRECI(I)*TON
      IF(I.GT.1) NR=(PRECI(I-1)+PRECI(I))*TON/2.
      IF(NR.LT.1) NR=1
      DTV=DTV/FLOAT(NR)
      PREK=PREC/FLOAT(NR)
      EK=EP/FLOAT(NR)
      IF(KP.EQ.NP) WRITE(6,57) DTV,PREK,EK
   57 FORMAT('0TIME STEP ',F6.3,'    RAIN ',F6.3,'    EVAP ',F6.3)
      DO 51 J=1,NR
      P1POS=P  (NDZ)
      IF(P1POS.LT.0.) P1POS=0.
      PREC=PREK
      EP=EK
C     EVAPORATION
      IF(PREC.NE.0) CALL BIX(X(7),EPX,PREC)
      EPC=0.
      IF(EPX.NE.0.) CALL BIX(EP,EPC,EPX)
      EP=EP-EPC
      EPC=0.
      CALL BIX(EP,EPO,DI)
      EP=EP-EPO
      EPC=EPC+EPO
      EMIN=(XK(P(NDZ),X(1),X(2),X(3))-X(9)*XK(0.,X(1),X(2),X(3)))/(XK(0.
     1,X(1),X(2),X(3))*(1.+AC-X(9)))*X(8)*FLOAT(NC)/FLOAT(NR*K)
      IF(EMIN.LT.0.) EMIN=0.
      IF(EP.GT.EMIN) EP=EMIN
      IF(EP.GT.X(8)*FLOAT(NC)/FLOAT(NR*K)) EP=X(8)*FLOAT(NC)/FLOAT(NR*K)
      EPC=EPC+EP
C     OVERLAND FLOW
      CALL FINDH(P  ,NDZ-1,H2,H3,DZ,0)
      EX2=0.
```

FIGURE A-2 (CONTINUED)

```
      NG=0
      FREC=.01
      BIL=0.
      RCUT=((H3/EX3)          )*X(1C)/1CC.*XK(0.,X(4),X(5),X(6))
      IF(P(NCZ).LE.C.) CI=0.
      IF(P(NCZ).LE.C.) GC TC 41
      IF(KP.EQ.NP) WRITE(6,43) CI,CELC
   43 FCRMAT('0',20X,'OVERLANC FLCW S/R LSEC    CI=',F8.3,'    CELC=',F8
     1.3)
      CALL GOFLOW(X(13),X(14),X(15),DI,CELC,CTV,BILL,AC)
      IF(KP.EQ.NP) WRITE(6,44) BILL,DI
   44 FCRMAT(' ',46X,'BILL=',F8.3,'    NEW CI=',F8.3)
C     SCIL MOISTURE CALC
   41 PREC=PREC-EP-EPC-BILL
      RIN=PREC/DTV
      IF(RIN.LT.0..ANC.P(NDZ).LT.-1.E5) GC TC 51
      CALL SCIL(P,    TH,RIN,ROLT,NCZ,DZ,CTV,X(1),X(2),X(3),X(4),X(5),X(
     16),NP,KP,  I,AC)
      P2PCS=P(NDZ)
      IF(P2PCS.LT.0.) P2PCS=0.
      GWF=RCUT*CTV
      DI=P2PCS
      DELC=(P2PCS-DI)/DTV
      TCUT=TCUT+GWF-PREC+P2POS-P1PCS
      TEP=TEP+EPC
      TBILL=TBILL+BILL
      TGWF=TGWF+GWF*(1.-X(11))
      GLOSS=GLOSS+GWF*X(11)
   51 Q(I)=C(I)+BILL+GWF*(1.-X(11))
      DC 39 J=1,NC
   39 Q(I-NC+J)=Q(I)/FLCAT(NC)
      PREC=0.
      EP=C.
      DTV=DT
      NC=1
      BILL=0.
      TIN=0.
      DC 42 J1=2,NDZ
   42 TIN=TIN+0.5*(TH(J1)+TH(J1-1))*DZ
      TCN=TTHIN-TIN
      TTH=TCN+EPXI-EPX-P2POS
      IF(PIN(NDZ).GT.0.) TTH=TTH+PIN(NDZ)
      DIF=TCUT-TCN
      IF(KP.EQ.NP) WRITE(6,10) TCUT,TCN,CIF
   10 FCRMAT('OCUMULATIVE NET OLTPUT =',F1C.3,'    MCISTURE DEPLETICN ='
     1,F10.3,'    ERRCR =',F10.6)
      IF(KP.EQ.NP) WRITE(6,36) LINE
    8 CCNTINUE
      IT=KLCCK(J)-IT
C
C     OUTPUT AND TICYUP STAGE
C
      CALL PFCRM(NDUM+1,NDT,QR,Q,E1,E2,E4,CCR,VAR,DET)
      IF(INDEX.NE.1.AND.INDEX.NE.11) GO TC 22
      QMA=QMAX
      PMA=PMAX
      IF(NPU.EQ.1) CALL PLUTO(NDT,C,QR,PRECI,FABEL,CMA ,PMA ,MS)
      IF(NPA.EQ.1) CALL PLOT(NDT,Q,QR,PRECI,FABEL,QMA,PMA,MS)
      SUMQ=TBILL+TGWF
      SIN=SUMP+TTH
      SOUT=SUMQ+TEP+GLOSS
      IF(SUMQ.LE.0.) GO TO 18
      TBILL=TBILL/SUMQ*100.
      TGWF=TGWF/SUMC*100.
      WRITE(6,19)
   19 FORMAT('OPERFCRMANCE SUMMARY'//)
      WRITE(6,59) E1,E2,E4,COR,VAR,DET
   59 FORMAT('OSUM OF THE ONETHS ',F8.3,'     SUM CF THE SQUARES',F1C.6
     1,'       SUM OF THE FOURTHS',F12.9/'OCOEFT CORRELATION ',F8.3,'
     2  COEFT OF VARIATION',F10.3,'      COEF DETERMINATION',F12.3/)
   18 WRITE(6,11) IT,SUMQR
   11 FCRMAT('OCALC TIME WAS',I5,'  SECONDS',10X,'TCTAL RECCRDED FLCW',F
     110.3/)
      WRITE(6,13)
   13 FORMAT(/' WATER BALANCE',67X,'FLOW ORIGIN IN PERCENT'/'OIN',40X,'C
     1UT')
      WRITE(6,14) SUMP,SUMQ,TBILL
   14 FORMAT('OTOTAL RAINFALL',6X,F10.3,9X,'TOTAL OLTFLOW',4X,F10.3,17X,
     1'RUNOFF   ',F5.1)
      WRITE(6,15) TTH,TEP,TGWF
```

FIGURE A-2 (CONTINUED)

```
15 FCRMAT('ODECREASE IN STCRAGE ',F10.3,9X,'TOTAL EVAPCTRANS ',F1C.3,
  116X,'THRUFLOW  ',F5.1)
   WRITE(6,16) GLOSS,SIN,SOUT
16 FCRMAT('O',39X,'GROUNDWATER LCSS ',F1C.3,25X,'------'//21X,'------
  1----',26X,'----------',26X, '100.0'/'C',2CX,F1C.3,26X,F1C.3)
   WRITE(6,17) NCRT,EPXI,DELCI,CII,SDI
17 FCRMAT(/'OMOISTURE STATUS     AT T =',I5,'     EPX',F8.3,'     DELC',
  1F8.3,'     DI',F8.3,'     SMOIST',F8.3)
   SD=EX3*EX1-TIN
   WRITE(6,17) NDT,EPX,DELD,DI,SD
   WRITE(6,36) LINE
   I2=0
   WRITE(6,34) I2,(J1,PIN(J1),J1=1,NDZ)
34 FCRMAT('OPRESSURE AT T =',I5,8(2X,I3,1X,F8.3)/(' ',20X,8(2X,I3,1X,
  1F8.3)))
   WRITE(6,35) I2,(J1,THIN(J1),J1=1,NDZ)
35 FCRMAT('OMOISTURE AT T =',I5,8(2X,I3,1X,F8.3)/(' ',20X,8(2X,I3,1X,
  1F8.3)))
   WRITE(6,36) LINE
   WRITE(6,34) NCT,(J1,P(  J1),J1=1,NDZ)
   WRITE(6,35) NDT,(J1,TH(J1),J1=1,NDZ)
   WRITE(6,36) LINE
   IF(INDEX+NPCH.EQ.12) WRITE(7,27) EPX,CELD,DI,(P(I),I=1,NCT)
27 FORMAT(3F5.2/(16F5.2))
   IF(INDEX+NPCH.EQ.3) WRITE(7,FMTF) (C(I),I=1,NCT)
22 CCNTINUE
24 CONTINUE
   RETURN
   END



   SUBROUTINE BIX(EPXM,EPX,PREC)
   EPXR=EPXM-EPX
   IF(PREC.GT.EPXR) GO TO 12
   EPX=EPX+PREC
   PREC=0.
   GC TO 13
12 EPX=EPXM
   PREC=PREC-EPXR
13 CCNTINUE
   RETURN
   END



   SUBROUTINE GOFLCW(EN,EL,ES,CI,DELD,CT,BILL,AC)
   BILL=0.
   IF(EN.LE.O..OR.EL.LE.O..OR.ES.LE.O.) GC TC 37
   D2=0.
   IF(CI+CELD*DT.LE.0.) GO TC 36
   IF(DELD.GT.0.) CE=.00981*(DELC*EN*EL/(    ES**.5))**.6
   IF(CELC.LE.0.) CE=0.
   K=2.*DT
   IF(K.LT.1) K=1
   DC 36 L=1,K
   N=0
   D2=CI+DT/FLOAT(K)*DELD
32 D1=D1+(D2-D1)/FLOAT(N+1)
33 N=N+1
   DAV=(DI+D1)/2.
   IF(DAV.LT.0.) DAV=0.
   GAY=1.
   IF(CE.GT.DAV) GAY=DAV/DE
   DB=0.
   IF(DAV.GT.0.) DB=1020. /(EN*EL*FLOAT(K))*DT*ES**.5*(DAV*(1.+.6*GAY
  1**3))**1.6667
   D2=CI+DT/FLOAT(K)*DELD-CB
   IF(N.GE.50 ) GO TC 138
   IF(D2.LT.0.)D1=D1/2.
   IF(D2.LT.0.)GC TO 33
   IF(ABS(D2-D1).GT.AC   )GO TO 32
138 IF(C2.LT.0.) CB=DI+DT/FLOAT(K)*DELC
   IF(D2.LT.0.)D2=0.
   BILL=BILL+DB
36 DI=D2
37 DELD=0.
   RETURN
   END
```

FIGURE A-2 (CONTINUED)

```
      SUBROUTINE SOIL(  PLL,TH,RIN,ROUT,NDZ,DZ,DT,A1,A2,A3,B1,B2,B3,NP,K
     1P,I2,AC)
      DIMENSION PLL(1),TH(1),A(100),B(100),C(100),D(100),P(100)
      DOUBLE PRECISION A,B,C,D,XK,CC
      NG=0
      FRED=1.
      EX2=0.
      RRIN=RIN
      P1POS=PLL(NDZ)
      IF(P1POS.LT.0.) P1POS=0.
      P1NEG=PLL(NDZ)
      IF(P1NEG.GT.0.) P1NEG=0.
      DO 78 I=1,NDZ
   78 P(I)=PLL(I)
      CALL FINDH(PLL,NDZ-1,H3,H6,DZ,0)
      IF(H6.GE.FLOAT(NDZ-3)*DZ) GO TO 44
      P4=(PLL(1)+PLL(2))/2.
      P5=(PLL(NDZ)+PLL(NDZ-1))/2.
   48 DO 79 I=1,NDZ
   79 PLL(I)=P(I)
      A(1)=1.
      B(1)=-1.
      C(1)=0.
      D(1) =DZ*(ROUT/XK(P4,B1,B2,B3)-1.)
      A(NDZ)=0.
      B(NDZ)=1.
      C(NDZ)=-1.
      D(NDZ)=DZ*(RIN/XK(P5,B1,B2,B3)-1.)
      DO 76 I6=3,NDZ
      P1=(PLL(I6)+PLL(I6-1))/2.
      P2=(PLL(I6-1)+PLL(I6-2))/2.
      P3=PLL(I6-1)
      A(I6-1)=XK(P1,B1,B2,B3)/DZ**2
      B(I6-1)=-1.*(CC(P3,A1,A2,A3)/DT+(XK(P1,B1,B2,B3)+XK(P2,B1,B2,B3))/
     1DZ**2)
      C(I6-1)=XK(P2,B1,B2,B3)/DZ**2
      D(I6-1)=(XK(P2,B1,B2,B3)-XK(P1,B1,B2,B3))/DZ-CC(P3,A1,A2,A3)*P3/DT
   76 CONTINUE
      CALL GE3(NDZ,1,A,B,C,D,PLL)
      PS=(PLL(NDZ)+PLL(NDZ-1))/2.
      QIN=XK(PS,B1,B2,B3)*(DZ+PLL(NDZ)-PLL(NDZ-1))/DZ
      PS=(PLL(2)+PLL(1))/2.
      QOUT=XK(PS,B1,B2,B3)*(DZ+PLL(2)-PLL(1))/DZ
      IF(NG.EQ.0.AND.PLL(NDZ).LE.0.) GO TO 28
C
C     STEPS FOR REDUCING RIN WHEN TOP BECOMES SATURATED
C
      NURG=1
      P2POS=PLL(NDZ)
      IF(P2POS.LT.0.) P2POS=0.
      EX1=RIN+(P2POS    -P1POS       )/DT-RRIN
      IF(ABS(EX1).LT.AC   .OR.NG.GT.49) GO TO 43
      IF((EX2.GE.0..AND.EX1.LT.0.).OR.(EX2.LT.0..AND.EX1.GE.0.)) FRED=FR
     1ED/5.
      NG=NG+1
      RIN=RIN-      FRED*ABS(EX1)/EX1
      EX2=EX1
      GO TO 48
C
C     STEPS FOR THE TOTALLY SATURATED CASE
C
   44 EX=(RIN-ROUT)*DT
      NURG=2
      PLL(NDZ)=PLL(NDZ)+EX
   50 P2POS=PLL(NDZ)
      IF(P2POS.LT.0.) P2POS=0.
      P2NEG=PLL(NDZ)
      IF(P2NEG.GT.0.) P2NEG=0.
      ZAV=0.5*(P1NEG+P2NEG)/(ROUT/XK(0.,B1,B2,B3)-1.)
      IF(ZAV.LT.0.) ZAV=0.
      DTH=ZAV*(XK(P2NEG,A1,A2,A3)-XK(P1NEG,A1,A2,A3))
      EX1=DTH-EX+P2POS-P1POS
      IF(ABS(EX1).LT.AC   .OR.NG.GT.49) GO TO 49
      IF((EX2.GE.0..AND.EX1.LT.0.).OR.(EX2.LT.0..AND.EX1.GE.0.)) FRED=FR
     1ED/5.
      EX2=EX1
      NG=NG+1
      PLL(NDZ)=PLL(NDZ)-FRED*ABS(EX1)/EX1
      GO TO 50
   49 DO 45 J1=2,NDZ
```

FIGURE A-2 (CONTINUED)

```
      J2=NDZ-J1+1
   45 PLL(J2)=PLL(J2+1)-(ROUT/XK(C.,B1,B2,B3)-1.)*DZ
      QIN=DTH/DT+ROUT
      QOUT=ROUT
      RIN=QIN
   43 IF(KP.EQ.NP)              WRITE(6,77) NURG,NG,EX1,FRED
   77 FORMAT('0',20X,'SOIL ITERATION    NURG=',I2,'    NG=',I3,'    EX1=
     1',F8.5,'    FRED=',F8.5)
   28 IF(KP.EQ.NP) WRITE(6,34) I2,(J1,PLL(J1),J1=1,NDZ)
   34 FORMAT('OPRESSURE AT T =',I5,8(I4,F10.3)/(' ',20X,8(I4,F10.3)))
      IF(KP.EQ.NP) WRITE(6,15) QIN,QOUT
   15 FORMAT('0',20X,'QIN =',F10.3,'    QOUT =',F10.3)
      IF(KP.EQ.NP) WRITE(6,17) RIN ,ROUT
   17 FORMAT(21X,'RIN =',F10.3,'    ROUT =',F10.3)
   38 DO 16 J1=1,NDZ
   16 TH(J1)=XK(PLL(J1),A1,A2,A3)
    1 CONTINUE
      RETURN
      END


      SUBROUTINE FINDH(P,NM,H1,H3,  DZ,NP)
      DIMENSION P(  100)
      H1=0.
      H3=0.
      DO 47 J1=1,NM
      J2=NM-J1+2
      IF(P(  J2-1).LT.0.E0.AND.P(  J2).GE.0.E0) H1=(J1-1+P(  J2)/(P(  J2
     1)-P(  J2-1)))*DZ
   47 IF(P(  J2-1).GE.0.E0.AND.P(  J2).LT.0.E0) H3=(J2-1-P(  J2)/(P(  J2
     1)-P(  J2-1)))*DZ
      IF(H3.EQ.0..AND.P(NM+1).GT.0..AND.P(1).GT.0.) H3=FLOAT(NM)*DZ+P(NM
     1+1)
      IF(NP.EQ.1 ) WRITE(6,55) H1,H3
   55 FORMAT(' TOP',F8.3,'    BOT',F8.3)
      RETURN
      END


      FUNCTION XK(P,A1,A2,A3)
      DOUBLE PRECISION XK
C     TANGENT 3-PARAMETER MOIST/PERM
      XK=90./A3*(1.+0.636*ATAN(A1/A2))
      IF(P.LT.0.) XK=90./A3*(1.+0.636* ATAN((P+A1)/A2))
      RETURN
      END


      FUNCTION CC(P,A1,A2,A3)
      DOUBLE PRECISION CC
C     TANGENT 3-PARAMETER MOIST-DERIV
      CC=0.
      IF(P.LT.0.) CC=57.3*A2/(A3*(A2**2+(A1+P)**2))
      RETURN
      END


      SUBROUTINE GE3(NDZ,NST,A,B,C,D,P)
      DIMENSION A(100),B(100),C(100),D(100),P(  100)
      DOUBLE PRECISION A,B,C,D
      DO 1 M=2,NDZ
      N=M+NST-1
      IF(C(N).EQ.0.) GO TO 1
      B(N)=-1.*B(N-1)*B(N)/C(N)+A(N-1)
      A(N)=-1.*B(N-1)*A(N)/C(N)
      D(N)=-1.*B(N-1)*D(N)/C(N)+D(N-1)
    1 CONTINUE
      P(  NDZ+NST)=0.
      DO 2 M=1,NDZ
      N=NDZ-M+NST
      D(N)=D(N)-A(N)*P(  N+1)
      IF(B(N).EQ.0.) WRITE(6,3) N,P(N)
    3 FORMAT('OB(N) = 0.0    N=',I5,'    P(N)=',F20.3/)
    2 P(  N)=D(N)/B(N)
      RETURN
      END
```

FIGURE A-2 (CONTINUED)

```
      SUBROUTINE PLUTO(ND,Q,QR,PRECI,FABEL,SM,PM,MS)
      DIMENSION QR(1),Q(1),PRECI(1)
      LOGICAL*1 LINE(132),BLANK,DOT,X,CIRCLE,AST,FABEL(80)
      DATA BLANK,DOT,X,CIRCLE,AST/' ','.','X','C','*'/
      I=0
      SUMQ=0.
      SUMQR=0.
      SUMP=0.
      DO 103 J=1,ND
      I=I+1
      SUMQ=SUMQ+Q(J)
      SUMQR=SUMQR+QR(J)
      SUMP=SUMP+PRECI(J)
      IF(I.LT.MS) GC TO 103
      IF(SUMQR.GT.SM) SM=SUMQR
      IF(SUMQ.GT.SM) SM=SUMQ
      IF(SUMP.GT.PM) PM=SUMP
      SUMQR=0.
      SUMQ=0.
      SUMP=0.
      I=I-MS
  103 CONTINUE
      IF(SM.EQ.0.) SM=1.
      IF(PM.EQ.0.) PM=1.
      WRITE(6,4) FABEL,SM,PM
    4 FCRMAT('0',80A1/'OGRAPH OF RECORDED FLOW(SYMBCL X) ANC SIMULATED F
     1LOW(SYMBOL O)',50X,'GRAPH OF RAINFALL'/'OMAX CRDINATE ',F8.4,' INC
     1HES PER TIME UNIT, DT',61X,'MAX ORD ',F8.4,' IN/DT'//)
      DO 101 J=1,132
  101 LINE(J)=DOT
      WRITE(6,3) LINE
      DO 102 J=2,131
  102 LINE(J)=BLANK
      I=0
      SUMQ=0.
      SUMQR=0.
      SUMP=0.
      DO 104 J=1,ND
      I=I+1
      SUMQ=SUMQ+Q(J)
      SUMQR=SUMQR+QR(J)
      SUMP=SUMP+PRECI(J)
      IF(I.LT.MS) GC TO 104
      J1=SUMQR/SM*108.+1.5
      J2=SUMQ/SM*108.+1.5
      LINE(J1)=X
      LINE(J2)=CIRCLE
      IF(J1.EQ.J2) LINE(J2)=AST
      J3=SUMP/PM*21.+111.5
      DO 105 J5=111,J3
  105 LINE(J5)=AST
      WRITE(6,3) LINE
    3 FCRMAT(' ',132A1)
      LINE(J1)=BLANK
      LINE(J2)=BLANK
      DO 106 J5=111,J3
  106 LINE(J5)=BLANK
      LINE(132)=DOT
      SUMQR=0.
      SUMQ=0.
      SUMP=0.
      I=I-MS
  104 LINE(1)=DOT
      DO 116 J=1,132
  116 LINE(J)=DOT
      WRITE(6,3) LINE
      RETURN
      END
```

FIGURE A-2 (CONTINUED)

```
      SUBROUTINE PLCT(NC2,Q,QR,PRECI,FABEL,QMAX,PMAX,MS)
C     THIS ROUTINE PLOTS OVER A FULL PAGE
C     NUMBER OF POINTS IS LIMITED TC 576
C
      DIMENSION Q(1),CR(1),PRECI(1),D(1152),E(1152)
      LOGICAL*1 FABEL(80),NAME4(10)/'TIME STEP '/,NAME5(30)/'FLOW *10  I
     1NCHES PER TIME STEP'/,NAME6(8)/'RAIN *1C'/,NAME7(9)/'SIMULATED'/,N
     2AME8(8)/'RECORDED'/
      CALL AINIT(ND2*3/MS+300)
      CALL AORIG(200,100)
      CALL ASCA(-80,-40,ND2*3/MS,0,1,ND2/MS-1,2,2,2)
      CALL ALAB(0,-80,NAME4,10,2,2)
      I=0
      SUMQ=0.
      SUMCR=0.
      SUMP=0.
      DO 1 K=1,ND2
      I=I+1
      SUMQ=SUMQ+Q(K)
      SUMQR=SUMQR+QR(K)
      SUMP=SUMP+PRECI(K)
      IF(I.LT.MS) GC TO 1
      IF(SUMQR.GT.QMAX) QMAX=SUMQR
      IF(SUMQ.GT.QMAX) QMAX=SUMC
      IF(SUMP.GT.PMAX) PMAX=SUMP
      SUMQR=0.
      SUMQ=0.
      SUMP=0.
      I=I-MS
    1 CONTINUE
      IF(QMAX.EQ.0.) QMAX=1.
      IF(PMAX.EQ.0.) PMAX=1.
      MM=0
      IF(QMAX.LT.1.) GO TC 7
    5 IF(QMAX.LE.10.) GO TO 6
      QMAX=QMAX/10.
      MM=MM-1
      GO TO 5
    7 IF(QMAX.GT.1.) GO TO 6
      QMAX=QMAX*10.
      MM=MM+1
      GO TO 7
    6 MM2=0
      IF(PMAX.LT.1.) GO TO 17
   15 IF(PMAX.LE.10.) GO TO 16
      PMAX=PMAX/10.
      MM2=MM2-1
      GO TO 15
   17 IF(PMAX.GT.1.) GO TO 16
      PMAX=PMAX*10.
      MM2=MM2+1
      GO TO 17
   16 NDIV1=QMAX+1.
      NDIV2=PMAX+1.
      IYINC1=700/NDIV1
      IYINC2=200/NDIV2
      CALL ABOX(0,0,1,NDIV1,ND2*3/MS,IYINC1,2)
      CALL ASCA(-120,-10,0,IYINC1,0,1,NDIV1+1,2,2)
      CALL ALAB(-60,0,NAME5,30,2,4)
      CALL ASCA(-80,130,0,1,MM,1,1,1,4)
      CALL ALAB(-110,0,FABEL,80,1,4)
      CALL ABOX(0,750,1,NDIV2,ND2*3/MS,IYINC2,2)
      CALL ASCA(-120,740,0,IYINC2,0,1,NDIV2+1,2,2)
      CALL ALAB(-60,750,NAME6,8,2,4)
      CALL ASCA(-80,880,0,1,MM2,1,1,1,4)
      K=0
      I=0
      SUMQ=0.
      DO 2 J=1,ND2
      I=I+1
      SUMQ=SUMQ+Q(J)
      IF(I.LT.MS) GC TO 2
      K=K+1
      D(K)=SUMQ*10.**MM/FLOAT(NDIV1)
      E(K)=K
      SUMQ=0.
      I=I-MS
    2 CONTINUE
      CALL ALINED(E,D,NC2/MS,1.,0.,33.3,0.143,2,4)
      K=0
```

FIGURE A-2 (CONTINUED)